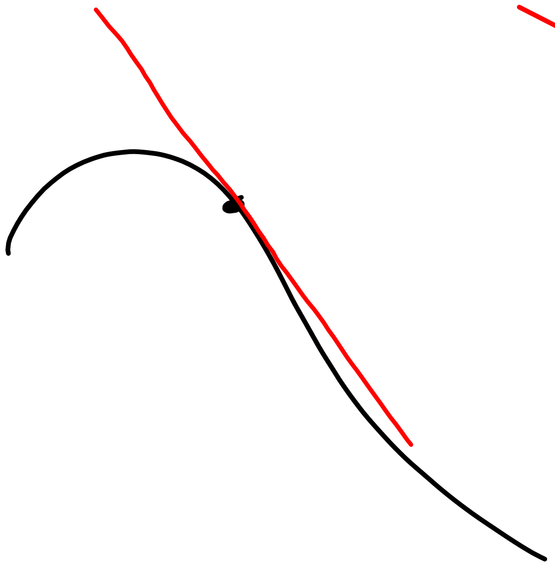
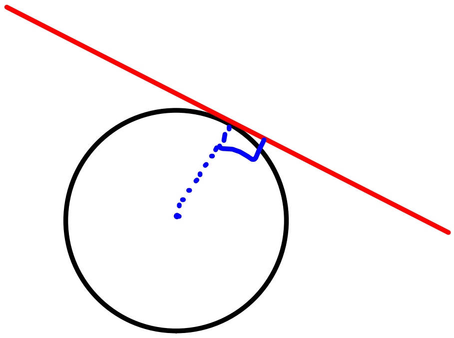




2.1 The Derivative & The Tangent Line Problem

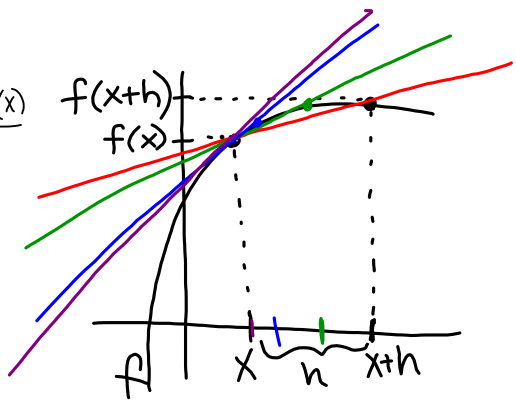
secant line crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

what happens as $h \to 0$?

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$



As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$f'(x)$	"f prime of x"
$\frac{dy}{dx}$	"derivative of y with respect to x"
y'	"y prime"
$\frac{d}{dx}[f(x)]$	"the derivative with respect to x of $f(x)$"
$D_x[y]$	"the partial derivative with respect to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

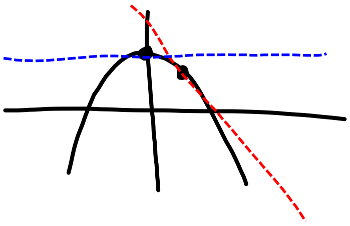
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\lim_{h \to 0} \frac{\sqrt[3]{8+h} - 2}{h}$$

$$= \left(\sqrt[3]{x}\right)' \Big|_{x=8}$$

8. $g(x) = 5 - x^2$

$g(2+h) = 5-(2+h)^2$



find slope of tangent line at the points $(2,1)$ & $(0,5)$

$(\underset{c}{2}, \underset{f(c)}{1})$

$$m = f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c)}{h}$$

@ $(2,1)$: $$g'(2) = \lim_{h\to 0} \frac{5-(2+h)^2 - 1}{h}$$

$$= \lim_{h\to 0} \frac{4-(4+4h+h^2)}{h} = \lim_{h\to 0} \frac{-4h-h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-4-h)}{\cancel{h}} = \boxed{-4}$$

@ $(0,5)$: $$g'(0) = \lim_{h\to 0} \frac{5-(0+h)^2-5}{h} = \lim_{h\to 0} \frac{-h^2}{h}$$

$$= \lim_{h\to 0} (-h) = \boxed{0}$$

20. $f(x) = x^3 + x^2$

find the derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 + (x+h)^2 - (x^3+x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{x^2} + 2xh + h^2 - \cancel{x^3} - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 + 2x + h)}{\cancel{h}} = \boxed{3x^2 + 2x}$$

Find the equation of the tangent line to

$f(x)=x^3-x$ at the point $(2,6)$.

$$m=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^3-(x+h)-(x^3-x)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x}-h-\cancel{x^3}+\cancel{x}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2-1)}{\cancel{h}}=3x^2-1=f'(x)$$

$$m=f'(2)=3(2)^2-1=11$$

$$y-y_1=m(x-x_1)$$

$$y-6=11(x-2)$$

$$\boxed{y=11x-16}$$

2.2 Basic Differentiation Rules

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

1. The <u>derivative of a constant function</u> is zero, i.e.,

for $c\in\mathbb{R}$, $\quad \frac{d}{dx}[c]=0$

<u>Proof:</u> $f(x)=c$

$$\lim_{h\to 0}\frac{c-c}{h}=\lim_{h\to 0}\frac{0}{h}=\lim_{h\to 0}0=\boxed{0}$$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

Special case: $\frac{d}{dx}[x] = 1$

$\frac{d}{dx}(x^1) = 1 \cdot x^0 = 1$

Proof:

Recall the binomial expansion:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + \frac{n!}{k!\,(n-k)!}a^{n-k}b^k + \cdots + b^n$$

Examples:

$\frac{d}{dx}[x^7] = 7x^6$

$\frac{d}{dx}[\pi^3] = 0$

$\frac{d}{dx}[2e] = 0$

$\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}\left[x^{1/2}\right] = \frac{1}{2}x^{\frac{1}{2}-1} = \boxed{\frac{1}{2}x^{-1/2}} = \boxed{\frac{1}{2\sqrt{x}}}$

$= \frac{1}{2x^{1/2}}$

$\frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}\left[x^{-3}\right]$

$= \boxed{-3x^{-4}} = \boxed{\frac{-3}{x^4}}$

$\frac{x^m}{x^n} = x^{m-n}$

$= \frac{1}{x^{n-m}}$

$\sqrt[m]{x^n} = x^{n/m}$

$x^{-n} = \frac{1}{x^n}$