$$\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5} \cdot \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3} = \lim_{x\to 5} \frac{\overbrace{x+4-9}^{\cancel{x-5}}}{\cancel{(x-5)}(\sqrt{x+4}+3)}$$

$$= \frac{1}{\sqrt{5+4}+3} = \frac{1}{6}$$

$$\lim_{\substack{\cancel{\Delta x}\to 0 \\ h}} \frac{f(x+\cancel{\Delta x}^{\,h}) - f(x)}{\underset{h}{\cancel{\Delta x}}}$$

$f(x) = 4x-3$

$f(x+h) = 4(x+h)-3$

$$= \lim_{h\to 0} \frac{4(x+h)-3-(4x-3)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4x}+4h\cancel{-3}\cancel{-4x}\cancel{+3}}{h}$$

$$= \lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}} = \boxed{4}$$

$f(x) = x^2 - 2x + 8$, $[2, 6]$, $f(c) = 11$

IVT applies $f(2) < 11$
$f(6) > 11$

$x^2 - 2x + 8 = 11$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$, $x = -1$

$$\lim_{x \to 5^-} \frac{-2|5-x|}{x-5}$$

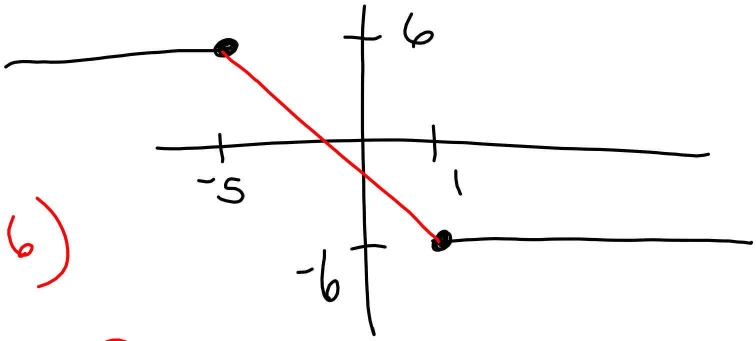
$$\frac{-2|5-x|}{x-5} = \begin{cases} \dfrac{-2\overbrace{(5-x)}^{-1(x-5)}}{x-5} = \boxed{2} & 5-x > 0,\ 5 > x,\ x < 5 \\ \dfrac{(-2)\cdot(-1)(5-x)}{x-5} = -2 & 5-x < 0,\ x > 5 \end{cases}$$

$-x > -5$

$x < 5$

$$6 = a(-5) + b$$
$$-(-6 = a(1) + b)$$
$$12 = -6a$$
$$-2 = a$$

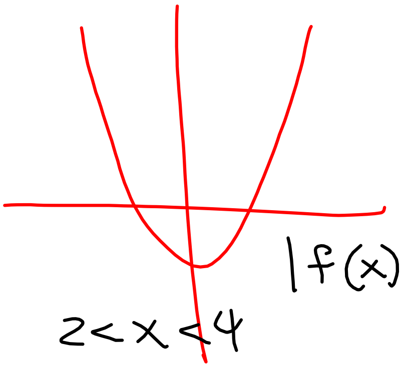
$$-6 = -2(1) + b$$
$$-4 = b$$



$(-5, 6)$ & $(1, -6)$

$$\frac{-6-6}{1-(-5)} = \frac{-12}{6} = -2$$
$$y-(-6) = -2(x-1)$$
$$y = -2x-4$$

$$f(x) = x^2 - 2$$



Find δ s.t. if $0 < |x - 3| < \delta$, then $|f(x) - 7| < \boxed{0.2} \leftarrow \varepsilon$

c, L

$$\lim_{x \to 3} f(x) = 7$$

$$|f(x) - L| = |x^2 - 2 - 7| = |x^2 - 9|$$
$$= |(x-3)(x+3)| < 7|x-3| < \varepsilon$$

need to replace w/ constant

$$|x-3| < \frac{0.2}{7} = \delta$$

works

$$\delta \approx 0.02857$$

$2 < x < 4$

$5 < x+3 < 7$

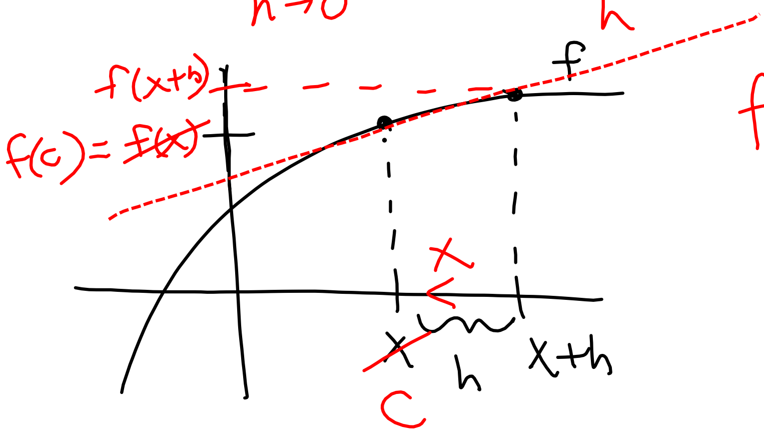
$2.9 < x < 3.1$

$x + 3 < 6.2$

$$\delta = \frac{0.2}{6.1} \approx 0.0327$$
$$\delta = \frac{0.2}{6.01} \approx 0.0332 \leftarrow$$

$$|f(x) - L| = \dots = k|x - c|$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

$$= \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f(x) = \sqrt[3]{x}$$

$2 = \sqrt[3]{8}$

$$f'(8) = \lim_{x \to 8} \frac{\sqrt[3]{x} - 2}{x - 8}$$

2.1 Differentiability & Continuity

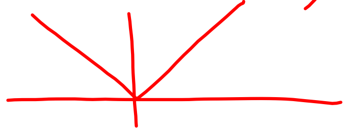
Alternative definition of the derivative at the point $(c, f(c))$:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

All differentiable functions are continuous, but not all continuous functions are differentiable.

e.g. $f(x) = |x|$

$$f'(0) = \lim_{x \to 0} \frac{|x| - |0|}{x - 0} = \lim_{x \to 0} \frac{|x|}{x}$$



$$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{\cancel{x}^{\,1}}{\cancel{x}} = 1$$

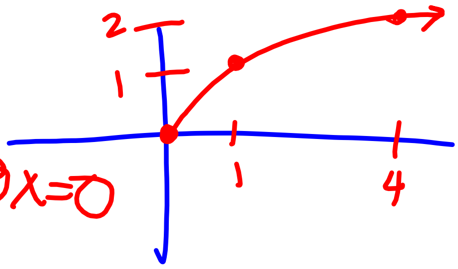
$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{\cancel{-x}}{\cancel{x}} = -1$$

Slope from the left & right are different
$\Rightarrow$ f has a sharp point

Because the left & right-hand limits are different, the limit in general and the derivative defined by it DO NOT EXIST

$f(x) = \sqrt{x}$

To show why $\sqrt{x}$ is not differentiable @ $x=0$



$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(0) = \lim_{x \to 0} \frac{\sqrt{x} - \sqrt{0}}{x - 0} = \lim_{x \to 0} \frac{\sqrt{x}}{x} = \lim_{x \to 0} \frac{x^{1/2}}{x^1}$$

$$= \lim_{x \to 0} \frac{1}{x^{1 - 1/2}}$$

$$\frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$$

We want only a single instance of x and no negative exponents

$$= \lim_{x \to 0^+} \frac{1}{\sqrt{x}} = \infty$$

$\sqrt{x}$ has a vertical tangent line @ $x = 0$