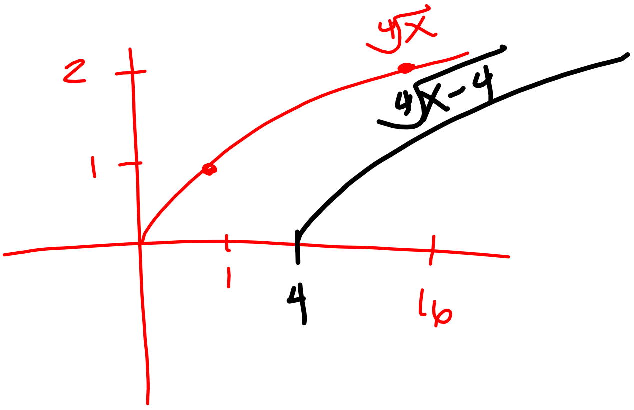
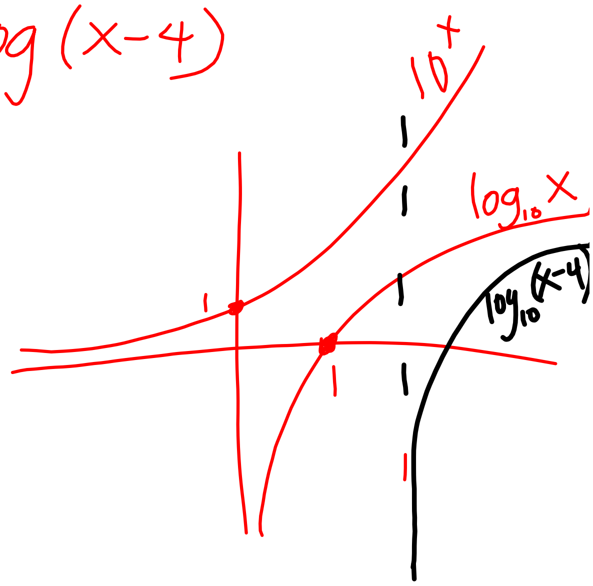
$y = \sqrt[4]{x-4}$ $\qquad y = \log(x-4)$





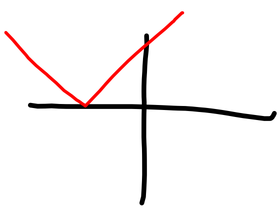
Use the alternate definition of the derivative to show that f is not differentiable at x = -3.

$f(x) = |x+3|$ $\rightarrow f'(c) = \lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$

$$\lim_{x \to -3} \frac{|x+3| - |-3+3|}{x-(-3)} = \lim_{x \to -3} \frac{|x+3|}{x+3}$$

$$\left.\begin{aligned} \lim_{x \to -3^-} \frac{|x+3|}{x+3} &= -1 \\ \lim_{x \to -3^+} \frac{|x+3|}{x+3} &= 1 \end{aligned}\right\}$$

since the left- & right-hand limits are different, the limit in general is undefined. Hence the derivitive defined by that limit does not exist



2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e.,

for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \in \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

$\frac{d}{dx}[x] = 1$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

$\frac{d}{dx}[cx] = c$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Examples:

$f(x) = 3x^2$

$f'(x) = 3(x^2)' = 3(2x) = \boxed{6x}$

$f(x) = \frac{3}{x} = 3x^{-1}$

$f'(x) = 3(x^{-1})' = 3(-1x^{-1-1}) \boxed{-3x^{-2}} = \boxed{\frac{-3}{x^2}}$

$g(x) = 2x^3 - x^2 + 3x$

$g'(x) = \boxed{6x^2 - 2x + 3}$

$y = 4x^{3/2} - 5x^4 + 2x^{\frac{1}{3}} - 7$

$y' = 4\left(\frac{3}{2}x^{3/2-2/2}\right) - 5(4x^3) + 2\left(\frac{1}{3}x^{1/3-3/3}\right) - 0$

$= \boxed{6x^{1/2} - 20x^3 + \frac{2}{3}x^{-2/3}}$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Proof that (sin x)'=cos x

$$\frac{a+b+c}{d} = \frac{a}{d} + \frac{b+c}{d}$$

$$\frac{d}{dx}[\sin x] = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\frac{-\sin x + \sin x \cos h}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[-\sin x \frac{(1 - \cos h)}{h} + \cos x \cdot \frac{\sin h}{h} \right]$$

$$= (-\sin x)(0) + (\cos x)(1) = \boxed{\cos x}$$

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e., for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

$\frac{d}{dx}[x] = 1$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

$\frac{d}{dx}[cx] = c$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

2.2

22. $y = 5 + \sin x$

$y' = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8x^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$

$y' = \boxed{-\frac{15}{8}x^{-4} - 2\sin x}$

$= -\frac{15}{8x^4} - 2\sin x$

44. $h(x) = \frac{2x^3 - 3x + 1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + x^{-1}$

$h'(x) = \boxed{4x - x^{-2}} = 4x - \frac{1}{x^2} = \frac{4x \cdot x^2}{x^2} - \frac{1}{x^2} = \frac{4x^3 - 1}{x^2}$

46. $y = 3x(6x - 5x^2) = 18x^2 - 15x^3$

$y' = 36x - 45x^2$