The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e., for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

$\frac{d}{dx}[x] = 1$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

$\frac{d}{dx}[cx] = c$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x$

$[x^n]' = nx^{n-1}$

$= 2x^{-1/3} + 3\cos x$

$$f'(x) = \boxed{-\frac{2}{3}x^{-4/3} - 3\sin x}$$

2.2 cont.

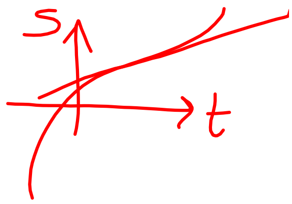
$s(t) = $ position — m, ft, km, mi, cm, yd

$V(t) = s'(t) = $ velocity — m/s, mi/h

$a(t) = V'(t) = s''(t) = $ acceleration — m/s^2

distance = rate × time

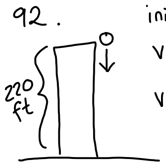
$\frac{\text{distance}}{\text{time}} = $ rate



average velocity: $\frac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

92.



initial velocity $V_0 = -22$ ft/s

$v(3) = ? = -32(3) - 22 = \boxed{-118 \text{ ft/s}}$

$v(t) = ?$ after falling 108 ft

$= -32(2) - 22 = \boxed{-86 \text{ ft/s}}$

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

position as a function of time; acceleration; initial velocity; initial position

$g = -9.8\ m/s^2$

$= -32\ ft/s^2$

$S(t) = \frac{1}{2}(-32)t^2 - 22t + 220$

$S(t) = -16t^2 - 22t + 220$

$V(t) = S'(t) = -32t - 22$

$-16t^2 - 22t + 220 = 108$

$0 = 16t^2 + 22t - 112$

$\frac{0}{2} = \cancel{2}(8t^2 + 11t - 56)$

$t = \frac{-11 \pm \sqrt{11^2 - 4(8)(-56)}}{2(8)}$

$= \frac{-11 \pm \sqrt{121 + 1792}}{16}$

$= \frac{-11 \pm \sqrt{1913}}{16}$

≈ 2 s

56
112
224
448
896
1792

The volume of a sphere is given by $V(r) = \frac{4}{3}\pi r^3$

Find the rate of change of volume with respect to radius when the radius is 2 cm.

$$\frac{d}{dr}[V(r)] = V'(r) = \frac{4}{3}\pi(3r^2)$$

$$= 4\pi r^2 \quad = \text{surface area of a sphere}$$

$$V'(2) = 4\pi(2)^2 = \boxed{16\pi\ cm^2}$$

What is the average rate of change of volume as r changes from 1cm to 3cm?

$$\frac{\Delta V}{\Delta r} = \frac{\frac{4}{3}\pi(3)^3 - \frac{4}{3}\pi(1)^3}{3-1}$$

$$= \frac{36\pi - \frac{4}{3}\pi}{2} = 18\pi - \frac{2}{3}\pi = \boxed{\frac{52\pi}{3}\ cm^2}$$

Power Rule:
$$\frac{d}{dx}[x^n] = nx^{n-1} \qquad d/dx\,[c]=0$$

Constant Multiple Rule:
$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Sum & Difference:
$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Product Rule:
$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule:
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Chain Rule:
$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

2.3 Product & Quotient Rules

$$[fg]' = \frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\boxed{(fg)' = f'g + fg'}$$

$$\left[\frac{f}{g}\right]' = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

"$\underbrace{\text{low}}_{}^{g(x)}$ $\overbrace{\text{dee high}}^{f'(x)}$ less $\overbrace{\text{high}}^{f(x)}$ $\overbrace{\text{dee low}}^{g'(x)}$, draw the line and square below $[g(x)]^2$"

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

$$[f(x)]^2 = f^2(x)$$

$$f(x)^2 = f(x^2)$$

2.3

6. $g(x) = (\sqrt{x})(\sin x) = (x^{1/2})(\sin x)$

$$g'(x) = (x^{1/2})'(\sin x) + (x^{1/2})(\sin x)'$$

$$= \boxed{\left(\tfrac{1}{2}x^{-1/2}\right)(\sin x) + (x^{1/2})(\cos x)}$$

$$= \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$$

12. $f(t) = \dfrac{\cos t}{t^3} = (\cos t)(t^{-3})$

$f'(t) = \sin t(t^{-3}) + (\cos t)(-3t^{-4})$

$$f'(t) = \frac{(t^3)(\cos t)' - (\cos t)(t^3)'}{(t^3)^2}$$

$$= \boxed{\frac{t^3(-\sin t) - (\cos t)(3t^2)}{t^6}}$$

$$= \frac{\cancel{t^2}(-t\sin t - 3\cos t)}{t^{\cancel{6}\,4}}$$

$$= \boxed{\frac{-t\sin t - 3\cos t}{t^4}}$$

26. $f(x) = \dfrac{x^3+3x+2}{x^2-1}$

Note: as a product,

$f(x) = (x^3+3x+2)(x^2-1)^{-1}$

we don't know how to differentiate this yet so we have to use the quotient rule!