Differential Calculus - Derivatives (Ch 2 & 5)

April 09, 2018

Power Rule:
$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Product Rule:
$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Constant Multiple Rule:
$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Quotient Rule:
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Sum & Difference:
$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Chain Rule:
$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$
$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Exponential and Logarithmic Functions:

$$\frac{d}{dx}[a^x] = a^x \ln a \qquad \frac{d}{dx}[e^x] = e^x$$
$$\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a} \qquad \frac{d}{dx}[\ln x] = \frac{1}{x}$$

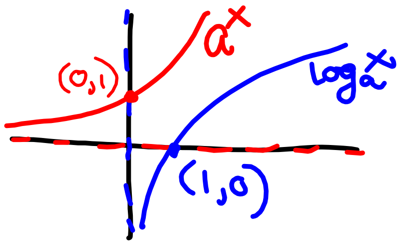
Ch 5 - Derivatives of Logarithmic and Exponential Functions

recall: $\ln x = \log_e x$

$e \approx 2.7$

$\log_2 8 = 3 \iff 2^3 = 8$

$\log_a b = c \iff a^c = b$



$y = 2^x$

x = the power to which we raise 2 to get y

= the # of times we multiply 2 by itself to get y

$= \log_2 y$

$$\frac{d}{dx}\left[2^x\right] = 2^x \ln 2$$

$$\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a$$

$$\frac{d}{dx}\left[\log_2 x\right] = \frac{1}{x \ln 2}$$

$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x \cdot \ln a}$$

$$a^x \neq x^n$$

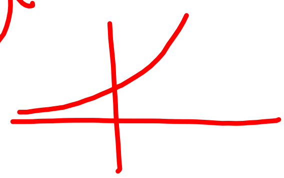
$$\left[x^n\right]' = nx^{n-1}$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

$$\left[e^x\right]' = e^x \cdot \ln e = e^x \log_e e = e^x$$

$$\left[e^x\right]' = e^x$$



$$\left[\ln x\right]' = \frac{1}{x \ln e} = \frac{1}{x}$$

Since the derivative of e^x is itself, this means that graphically, at every x-value, the slope of the tangent line at that point is exactly the y-coordinate.

$$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$$

$$\frac{d}{dx}\left[5^{\sin x}\right] = 5^{\sin x} \cdot \ln 5 \cdot \cos x$$

$$\frac{d}{dx}\left[\ln(f(x))\right] = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}\left[\log_5(\sin x)\right] = \frac{1}{(\sin x)(\ln 5)} \cdot \cos x = \frac{\cos x}{(\sin x)\ln 5}$$

$$= \frac{\cot x}{\ln 5}$$

$$f(x) = \cos\left(\sqrt{\tan^2 x - 2x}\right) = \cos\left(\left[(\tan x)^2 - 2x\right]^{1/2}\right)$$

$$f'(x) = \left[-\sin\left(\sqrt{\tan^2 x - 2x}\right)\right] \cdot \frac{1}{2}\left[\tan^2 x - 2x\right]^{-1/2} \cdot$$

$$\cdot \left(2\tan x \sec^2 x - 2\right)$$

1. $f(x) = \cot(5x^2 - 3x)$

$$f'(x) = \left[-\csc^2(5x^2-3x)\right] \cdot (10x-3)$$

2. $f(x) = \sqrt[3]{\csc(4x)} = \left[\csc(4x)\right]^{1/3}$

$$f'(x) = \left[\frac{1}{3}\left[\csc(4x)\right]^{-2/3}\right] \cdot \left(-\csc(4x)\cot(4x)\right) \cdot 4$$

3. $f(x) = \frac{\sin 2x}{x^3} = (\sin 2x)(x^{-3})$

$$f'(x) = \frac{x^3(\cos 2x \cdot 2) - (\sin 2x) \cdot 3x^2}{(x^3)^2}$$

$$= \frac{x^2(2x\cos 2x - 3\sin 2x)}{x^6}$$

$$= \boxed{\frac{2x\cos 2x - 3\sin 2x}{x^4}}$$

$f(x) = \ln[\sin(5x^3+2x)]$

$$f'(x) = \frac{1}{\sin(5x^3+2x)} \cdot \cos(5x^3+2x) \cdot (15x^2+2)$$

$$= (15x^2+2)\cot(5x^3+2x)$$

$$= \frac{15x^2+2}{\tan(5x^3+2x)}$$

$$f(x) = (\sec x)(5^{\sin x})$$

$$f'(x) = (\sec x \tan x)(5^{\sin x}) + (\cancel{\sec x})(5^{\sin x} \ln 5 \cdot \cancel{\cos x})$$

$$f(x) = \frac{(x^2 \ln x)}{\sin x}$$

$$f'(x) = \frac{(\sin x)(x^2 \ln x)' - (x^2 \ln x)(\sin x)'}{(\sin x)^2}$$

$$= \frac{(\sin x)(2x \ln x + x^2 \cdot \frac{1}{x}) - (x^2 \ln x)(\cos x)}{\sin^2 x}$$

$$= \frac{2x \ln x \sin x + x \sin x - x^2 \ln x \cos x}{\sin^2 x}$$

$[x^n]' = nx^{n-1}$

$[cf(x)]' = cf'(x)$

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$

$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

$[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$[e^x]' = e^x$

$[a^x]' = a^x \ln a$

$[\ln x]' = \frac{1}{x}$

$[\log_a x]' = \frac{1}{x \ln a}$

$[\sin x]' = \cos x$

$[\cos x]' = -\sin x$

$[\tan x]' = \sec^2 x$

$[\cot x]' = -\csc^2 x$

$[\sec x]' = \sec x \tan x$

$[\csc x]' = -\csc x \cot x$

$[\arcsin x]' = \frac{1}{\sqrt{1-x^2}}$

$[\arctan x]' = \frac{1}{1+x^2}$

$[\operatorname{arcsec} x]' = \frac{1}{|x|\sqrt{x^2-1}}$

$[\arccos x]' = \frac{-1}{\sqrt{1-x^2}}$

$[\operatorname{arccot} x]' = \frac{-1}{1+x^2}$

$[\operatorname{arccsc} x]' = \frac{-1}{|x|\sqrt{x^2-1}}$

$$\arcsin x = \sin^{-1}(x) \neq \frac{1}{\sin x}$$

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

5.8

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$
$$\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\operatorname{arccot} x] = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}[\operatorname{arccsc} x] = \frac{-1}{|x|\sqrt{x^2-1}}$$

44. $f(x) = \operatorname{arcsec} 2x$

$$f'(x) = \frac{1}{|2x|\sqrt{(2x)^2-1}} \cdot 2 = \frac{1}{|x|\sqrt{4x^2-1}}$$

48. $h(x) = (x^2)\arctan x$

$$h'(x) = 2x\arctan x + \frac{x^2}{1+x^2}$$

52. $y = \ln(t^2+4) - \frac{1}{2}\arctan\frac{t}{2}$

$$(t/2) = \frac{1}{2}t$$

$$y' = \frac{1}{t^2+4} \cdot 2t - \frac{1}{2}\,\frac{1}{1+(t/2)^2} \cdot \frac{1}{2}$$

$$= \frac{2t}{t^2+4} - \frac{1}{4} \cdot \frac{1}{(1+t^2/4)} = \frac{2t}{t^2+4} - \frac{1}{4+t^2}$$

$$= \boxed{\frac{2t-1}{t^2+4}}$$