19. $f(x) = 9 - 5x^2 + 4x^{1/3} + x^{-3}$

$f'(x) = \boxed{-10x + \frac{4}{3}x^{-2/3} - 3x^{-4}}$

20. $f(x) = (x^5)(\sin x)$

$f'(x) = \boxed{5x^4 \sin x + x^5 \cos x}$

21. $f(x) = \dfrac{\cos x}{7x-5}$

$f'(x) = \boxed{\dfrac{(7x-5)(-\sin x) - (\cos x)(7)}{(7x-5)^2}}$

$= \dfrac{(5-7x)\sin x - 7\cos x}{(7x-5)^2}$

22. $f(x) = \dfrac{1}{x^3} + \sqrt{x} - \dfrac{1}{\sqrt[3]{x}}$

$= x^{-3} + x^{1/2} - x^{-1/3}$

$f'(x) = \boxed{-3x^{-4} + \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-4/3}}$

$= \dfrac{-3}{x^4} + \dfrac{1}{2\sqrt{x}} + \dfrac{1}{3\sqrt[3]{x^4}}$

23. $f(x) = \sin(5x)$

$f'(x) = \boxed{[\cos(5x)] \cdot 5} = \boxed{5\cos 5x}$

$$f(x) = 5^{\sqrt[3]{4\log_2(3x^2-4x)}} = 5^{(4\log_2(3x^2-4x))^{1/3}}$$

$$f'(x) = \boxed{5^{\sqrt[3]{4\log_2(3x^2-4x)}} \cdot \ln 5 \cdot \frac{1}{3}(4\log_2(3x^2-4x))^{-2/3} \cdot \frac{4}{(3x^2-4x)\ln 2} \cdot (6x-4)}$$

2.4 The Chain Rule, cont.

18. $f(x) = -3\sqrt[4]{2-9x}$ $= -3(2-9x)^{1/4}$

$$f'(x) = -\frac{3}{4}(2-9x)^{-3/4} \cdot (-9) = \boxed{\frac{27}{4\sqrt[4]{(2-9x)^3}}}$$

32. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2$ $= \frac{t^4}{(t^3+2)^2}$

$$h'(t) = \frac{(t^3+2)^2 \cdot 4t^3 - t^4 \cdot 2(t^3+2) \cdot 3t^2}{(t^3+2)^4} = \frac{2t^3\cancel{(t^3+2)}\left[2(t^3+2) - 3t^3\right]}{(t^3+2)^{\cancel{4}\,3}}$$

$$= \cancel{\boxed{\frac{2t^3(2-3t^3)}{(t^3+2)^3}}}$$

50. $h(x) = \sec x^2$

$$= \sec(x^2)$$

$$h'(x) = [\sec(x^2)\tan(x^2)] \cdot 2x$$

$$= \boxed{2x\sec x^2 \tan x^2}$$

$$\sec^2 x = (\sec x)^2$$

$$\sec x^2 = \sec(x^2)$$

60. $g(t) = 5\cos^2 \pi t$ $= 5(\cos\pi t)^2$

$\sin 2x = 2\sin x\cos x$
$\sin 2\pi t = 2\sin\pi t\cos\pi t$

$g'(x) = (10\cos\pi t)\cdot(-\sin\pi t)\cdot\pi = -10\pi\sin\pi t\cos\pi t$
$= \boxed{-5\pi\sin 2\pi t}$

66. $y = \sin\sqrt[3]{x} + \sqrt[3]{\sin x}$
$= \sin(x^{1/3}) + (\sin x)^{1/3}$

$y' = [\cos(x^{1/3})]\cdot(\frac{1}{3}x^{-2/3}) + \frac{1}{3}(\sin x)^{-2/3}\cdot\cos x$

5.4

46. $g(t) = e^{-3/t^2}$ $= e^{-3t^{-2}}$

$g'(t) = (e^{-3t^{-2}})(6t^{-3}) = \boxed{\dfrac{6}{e^{3/t^2}t^3}}$

48. $y = \ln\left(\frac{1+e^x}{1-e^x}\right) = \ln(1+e^x) - \ln(1-e^x)$

$y' = \boxed{\dfrac{e^x}{1+e^x} + \dfrac{e^x}{1-e^x}}$

$\log_a MN = \log_a M + \log_a N$
$\log_a \frac{M}{N} = \log_a M - \log_a N$
$\log_a(M^p) = p\log_a M$

58. $y = \ln e^x$ $= x$

$y' = \frac{1}{\cancel{e^x}}\cdot\cancel{e^x}$

$y' = \boxed{1}$

$\log_a a^x = x$
$a^{\log_a x} = x$

5.5

46. $f(t) = \frac{3^{2t}}{t}$

$$f'(t) = \frac{t(3^{2t})(\ln 3)(2) - (3^{2t})(1)}{t^2} = \frac{3^{2t}[2t\ln 3 - 1]}{t^2}$$

54. $y = \log_{10}\frac{x^2-1}{x} = \log_{10}(x^2-1) - \log_{10} x$

$$y' = \frac{2x}{(x^2-1)\ln 10} - \frac{1}{x\ln 10} = \frac{2x^2-(x^2-1)}{x(x^2-1)\ln 10}$$

$$= \boxed{\frac{x^2+1}{x(x^2-1)\ln 10}}$$

Find the second derivative.

82. $f(x) = \sec^2 \pi x = [\sec(\pi x)]^2$

$$f'(x) = (2\sec \pi x)\cdot(\sec \pi x \tan \pi x)\cdot \pi$$

$$= (2\pi \tan \pi x)(\sec^2 \pi x)$$

$$f''(x) = [(2\pi \sec^2 \pi x)(\pi)]\cdot[\sec^2 \pi x] + [2\pi \tan \pi x]\cdot[2\pi \tan \pi x \sec^2 \pi x]$$

$$= \boxed{2\pi^2 \sec^4 \pi x + 4\pi^2 \sec^2 \pi x \tan^2 \pi x}$$

5.1

58. $f(x) = \ln \sqrt[3]{\dfrac{x-1}{x+1}}$ $= \ln\left(\dfrac{x-1}{x+1}\right)^{1/3} = \frac{1}{3}\ln\dfrac{x-1}{x+1}$

$$= \frac{1}{3}\left[\ln(x-1) - \ln(x+1)\right]$$

$$= \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+1)$$

$$f'(x) = \frac{1}{3(x-1)} - \frac{1}{3(x+1)}$$

$$= \frac{x+1-(x-1)}{3(x^2-1)} = \boxed{\frac{2}{3(x^2-1)}}$$

1. $f(x) = \cos(5x)$

$$f'(x) = \boxed{-5\sin 5x}$$

2. $f(x) = \ln(\tan x)$

$$f'(x) = \frac{\sec^2 x}{\tan x} = \frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x \cos x} = \boxed{\csc x \sec x}$$

3. $f(x) = e^{\arctan 2x}$

$$f'(x) = e^{\arctan 2x} \cdot \frac{1}{1+(2x)^2} \cdot 2 = \boxed{\frac{2e^{\arctan 2x}}{1+4x^2}}$$

4. $f(x) = \sqrt[3]{\sec x}$ $= (\sec x)^{1/3}$

$$f'(x) = \frac{1}{3}\sec x^{-2/3} \cdot \sec x \tan x = \boxed{\frac{\tan x \sqrt[3]{\sec x}}{3}}$$

5. $f(x) = -4\sin(3e^x)$

$$f'(x) = -4\cos(3e^x) \cdot 3e^x$$

$$= \boxed{-12e^x \cos(3e^x)}$$