What happens if...

$x^2y + y^2x = -2$ ← implicit

explicit $y = f(x)$

$y = x^2 + 2x - \cos x$

how to find y'?

$y' = \frac{dy}{dx}$

$x' = \frac{dx}{dx} = 1$ $\quad \frac{d\theta}{d\theta} = 1$

$\frac{dy}{dy} = 1$ $\quad \frac{dt}{dt} = 1$

2.5 Implicit Differentiation

★ $y = f(x)$

y is a function of x

$\frac{d}{dx}[x] = 1$; $\frac{d}{dx}[y] = y'$

6. $x^2y + y^2x = 2$

$$\frac{d}{dx}\left[x^2y + y^2x\right] = \frac{d}{dx}\left[-2\right]$$

$y = f(x)$

$\frac{d}{dx}[y^2] = \frac{d}{dx}[f(x)]^2$

$2y \cdot y' = 2 \cdot f(x) \cdot f'(x)$

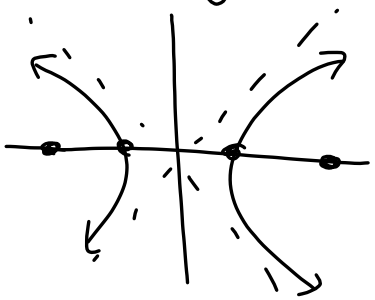
$$(2x)(y) + (x^2)y' + (2y \cdot y')x + y^2(1) = 0$$

$$x^2y' + 2xyy' = -2xy - y^2$$

$$y'(x^2 + 2xy) = -2xy - y^2$$

$$y' = \frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$

2. $x^2 - y^2 = 16$



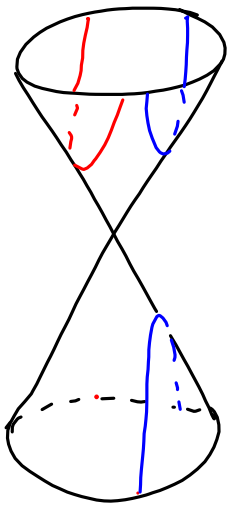
$$\frac{d}{dx}\left[x^2 - y^2\right] = \frac{d}{dx}\left[16\right]$$

$$2x - 2yy' = 0$$

$$2x = 2yy'$$

$$\frac{2x}{2y} = y'$$

$$\frac{dy}{dx} = \frac{x}{y}$$



8. $\sqrt{xy} = x - 2y$

$$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}\left[x - 2y\right]$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \left[1 \cdot y + x \cdot y'\right] = 1 - 2y'$$

$$2\sqrt{xy} \cdot \frac{y + xy'}{2\sqrt{xy}} = (1 - 2y') \cdot 2\sqrt{xy}$$

$$y + xy' = 2\sqrt{xy} - 4y'\sqrt{xy}$$

$$xy' + 4y'\sqrt{xy} = 2\sqrt{xy} - y$$

$$y'(x + 4\sqrt{xy}) = 2\sqrt{xy} - y$$

$$y' = \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}$$

10. $2\sin x \cos y = 1$

$$\frac{d}{dx}\left[2\sin x \cos y\right] = \frac{d}{dx}\left[1\right]$$

$$(2\cos x)(\cos y) + (2\sin x)(-y'\sin y) = 0$$

$$2\cos x \cos y = 2y' \sin x \sin y$$

$$\frac{2\cos x \cos y}{2\sin x \sin y} = y'$$

$$\cot x \cot y = y'$$

12. $(\sin\pi x + \cos\pi y)^2 = 2$

$\frac{d}{dx}\left[(\sin\pi x + \cos\pi y)^2\right] = \frac{d}{dx}[2]$

$2(\sin\pi x+\cos\pi y)(\pi\cos\pi x - \pi y'\sin\pi y) = 0$ ($2\sin\pi x + 2\cos\pi y$)

If $\sin\pi x \neq -\cos\pi y$, can divide both sides by $2(+)$

$2\pi\sin\pi x\cos\pi x - 2\pi y'\sin\pi x\sin\pi y + 2\pi\cos\pi x\cos\pi y - 2\pi y'\sin\pi y\cos\pi y = 0$

$\frac{2\pi\sin\pi x\cos\pi x + 2\pi\cos\pi x\cos\pi y}{2\pi\sin\pi x\sin\pi y + 2\pi\sin\pi y\cos\pi y} = y'$

$\frac{\sin\pi x\cos\pi x + \cos\pi x\cos\pi y}{\sin\pi x\sin\pi y + \sin\pi y\cos\pi y} = y'$

$\frac{\cos\pi x(\sin\pi x + \cos\pi y)}{\sin\pi y(\sin\pi x + \cos\pi y)} = y'$

$y = \frac{x^2}{x}$ vs. $y = x$