$$\frac{d}{dx}[y^3] = 3y^2 \cdot \frac{dy}{dx}$$

$$[y^3]' = 3y^2 y'$$

16. $x = \sec\frac{1}{y}$

$$\frac{d}{dx}[x] = \frac{d}{dx}\left[\sec(y^{-1})\right]$$

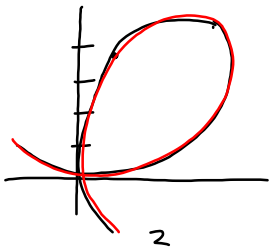
$$1 = \left(\sec\tfrac{1}{y}\tan\tfrac{1}{y}\right)\cdot(-y^{-2})\cdot y'$$

$$\frac{1}{-y^{-2}\sec\frac{1}{y}\tan\frac{1}{y}} = y'$$

$$y' = \boxed{-y^2\cos\tfrac{1}{y}\cot\tfrac{1}{y}}$$

32. Folium of Descartes $x^3+y^3-6xy=0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$



$\frac{d}{dx}[x^3+y^3] = \frac{d}{dx}[(6x)(y)]$

$3x^2+3y^2y' = 6y+6xy'$

$3y^2y'-6xy' = 6y-3x^2$

$y'(3y^2-6x) = 6y-3x^2$

$y' = \frac{6y-3x^2}{3y^2-6x} = \frac{3(2y-x^2)}{3(y^2-2x)}$

$y' = \frac{2y-x^2}{y^2-2x}$

$= \frac{2\left(\frac{8}{3}\right)-\left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2-2\left(\frac{4}{3}\right)}$

$= \frac{\frac{16}{3}-\frac{16}{9}}{\frac{64}{9}-\frac{8}{3}} = \frac{\frac{48-16}{9}}{\frac{64-24}{9}} = \frac{32}{40} = \boxed{\frac{4}{5}}$

40. Find y'' in terms of x & y.

$y^2=4x$

$\frac{d}{dx}[y^2] = \frac{d}{dx}[4x]$

$2yy' = 4$

$y' = \frac{4}{2y} = \frac{2}{y} = 2y^{-1}$

$\frac{d}{dx}[y'] = \frac{d}{dx}[2y^{-1}]$

$y'' = -2y^{-2}y'$

$y'' = \frac{-2y'}{y^2} = \frac{-2\left(\frac{2}{y}\right)}{y^2}$

$\boxed{y'' = \frac{-4}{y^3}}$

2.6 Related Rates

$\frac{dr}{dt} = 2 \text{ in/min}$

$\frac{dV}{dt} = ?$ when $r = 6$

18. $V = \frac{4}{3}\pi r^3$

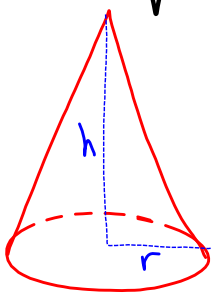
$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \left(\frac{4}{3}\pi\right)\left(3r^2 \cdot \frac{dr}{dt}\right)$$

$$= 4\pi(6)^2 \cdot 2$$

$$= \boxed{288\pi \ \frac{\text{in}^3}{\text{min}}}$$

22. $V = \frac{1}{3}\pi r^2 h$



$\frac{dr}{dt} = 2 \text{ in/min}$

$h = 3r \Rightarrow \frac{dh}{dt} = 3 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = ?$ when $r = 6\text{in}$

$h = 3(6) = 18\text{in}$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right]$$

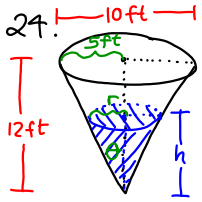
$$\frac{dV}{dt} = \frac{\pi}{3} \cdot 2r \cdot \frac{dr}{dt} \cdot h + \frac{\pi}{3} r^2 \cdot \frac{dh}{dt}$$

$$V = \frac{\pi}{3} r^2 (3r)$$

$$V = \pi r^3$$

$$\frac{dV}{dt} = 3\pi r^2 \cdot \frac{dr}{dt} = 3\pi(6)^2 \cdot 2 = \boxed{216\pi \ \frac{\text{in}^3}{\text{min}}}$$

24.



$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 10\ ft^3/min$

$\frac{dh}{dt} = ?$ when $h = 8$ ft

$\frac{r}{h} = \frac{5}{12}$

$r = \frac{5}{12}h$

$$V = \frac{\pi}{3}\left(\frac{5}{12}h\right)^2 h$$

$$V = \frac{25\pi}{3 \cdot 144}h^3$$

$$\frac{dV}{dt} = \left(\frac{25\pi}{3 \cdot 144}\right)\left(3h^2 \cdot \frac{dh}{dt}\right)$$

$$\frac{\frac{dV}{dt}}{\frac{25\pi}{144}h^2} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{144 \cdot \frac{dV}{dt}}{25\pi h^2} = \frac{144(10)}{25\pi(8)^2}$$

$$= \frac{3 \cdot 4 \cdot 3 \cdot 4 \cdot 2 \cdot 5}{5 \cdot 5 \cdot 2 \cdot 4 \cdot 2 \cdot 4 \pi} = \boxed{\frac{9}{10\pi}\ \frac{ft}{min}}$$