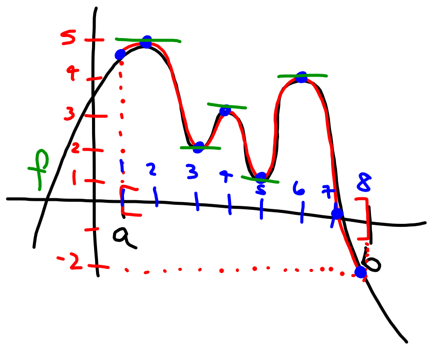
3.1 Extrema on an Interval

↳ maxima & minima

↳ relative & absolute



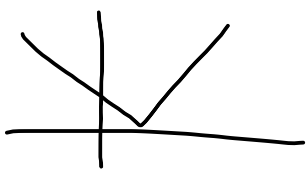
relative minima: (3,2), (5,1)

relative maxima: (2,5), (4,3), (6,4)

absolute maximum: 5 @ (2,5)

absolute minimum: -2 @ (8,-2)

$f(x)$ has a relative maximum or minimum when $f'(x) = 0$. or



$f'(x)$ is undefined.

We call such x-values

Critical #'s of f.

3.1 Find the absolute max & min on the closed interval.

28. $h(t) = \dfrac{t}{t-2}$, $[3,5]$

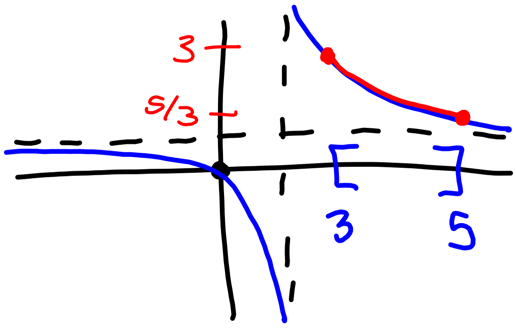
$h(3) = \dfrac{3}{3-2} = 3 \leftarrow$ abs. max

$h'(t) = \dfrac{(t-2)(1) - t(1)}{(t-2)^2}$

$h(5) = \dfrac{5}{5-2} = \dfrac{5}{3} \leftarrow$ abs. min.

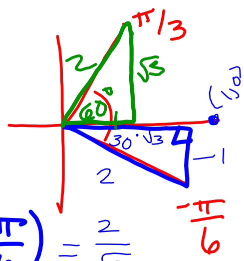
$h'(t) = \dfrac{-2}{(t-2)^2}$

critical #: 2



30. $g(x) = \sec x$, $\left[-\dfrac{\pi}{6}, \dfrac{\pi}{3}\right]$

Find the absolute max & min on the closed interval.



$g'(x) = \sec x \tan x$

$= \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x}$

$g'(x) = 0$ when $\sin x = 0$

$x = 0$ is a critical #

$g'(x)$ is undefined when $\cos x = 0$ contributes no critical #'s

$g\left(-\frac{\pi}{6}\right) = \sec\left(-\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$

$g(0) = \sec(0) = 1 \leftarrow$ abs. min.

$g\left(\frac{\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right) = 2 \leftarrow$ abs. max

$1 < 3 < 4$

$1 < \sqrt{3} < 2$

$1 > \frac{1}{\sqrt{3}} > \frac{1}{2}$

$2 > \frac{2}{\sqrt{3}} > 1$

$\dfrac{\cancel{a}}{\cancel{a}b} < \dfrac{\cancel{b}}{a\cancel{b}}$

22. $f(x) = x^3 - 12x$, $[0, 4]$

Find the absolute max & min on the closed interval.

$f'(x) = 3x^2 - 12$

$3(x^2 - 4) = 0$

$x = \pm 2$

critical # : 2

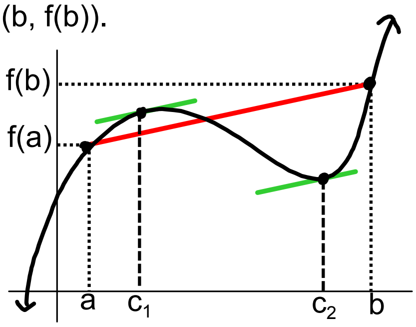
$f(0) = 0$

$f(2) = -16 \leftarrow$ abs. min

$f(4) = 16 \leftarrow$ abs. max.

3.2 Rolle's Theorem & The Mean Value Theorem

The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).



Is f(x) continuous on [a,b] & differentiable on (a,b)?

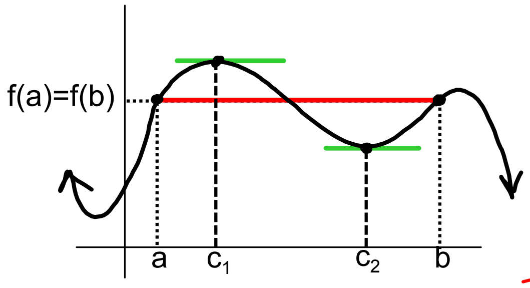
If so, then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

slope of tangent line @ (c, f(c))

slope of secant line through (a, f(a)) & (b, f(b))

Rolle's Theorem is a special case of the MVT where f(a)=f(b),
(and hence involving horizontal secant/tangent lines)



Criteria: f is continuous on $[a,b]$ & differentiable on (a,b), $f(a)=f(b)$

If so, Then $\exists$ (there exists) $c \in (a,b)$ s.t. $f'(c)=0$

Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x) = \frac{x+5}{x-2}\ , \qquad [1,3]$$

f is not continuous on [1,3].

$$g(x) = |x-2|\ , \qquad [1,3]$$

g is continuous on [1,3], but not differentiable on (1,3).