The radius of a cone is increasing at a rate of 3 centimeters per second and the height of the cone is *decreasing* at a rate of 4 centimeters per second.

At a certain instant, the radius is 8 centimeters and the height is 10 centimeters.

What is the rate of change of the volume of the cone at that instant (in cubic centimeters per second)?

Choose 1 answer:

- (A) $-\frac{224\pi}{3}$
- (B) $-\frac{736\pi}{3}$
- (C) $\frac{736\pi}{3}$
- (D) $\frac{224\pi}{3}$

The volume of a cone with radius r and height h is $\pi r^2 \frac{h}{3}$.

$V = \left(\frac{\pi}{3} r^2\right) h$; $\frac{dr}{dt} = 3 \frac{cm}{s}$; $\frac{dh}{dt} = -4 \frac{cm}{s}$

$\frac{dV}{dt} = ?$ when $r = 8cm$, $h = 10cm$

$$\frac{dV}{dt} = \frac{2\pi}{3} r \cdot \frac{dr}{dt} \cdot h + \frac{\pi}{3} r^2 \cdot \frac{dh}{dt}$$

The side of the base of a square pyramid is increasing at a rate of 6 meters per minute and the height of the pyramid is *decreasing* at a rate of 1 meter per minute.

At a certain instant, the base's side is 3 meters and the height is 9 meters.

What is the rate of change of the volume of the pyramid at that instant (in cubic meters per minute)?

Choose 1 answer:

- (A) -105
- (B) 111
- (C) -111
- (D) 105

The volume of a square pyramid with base side s and height h is $\frac{1}{3}s^2 h$.

$V = \frac{1}{3} s^2 h$

$\frac{ds}{dt} = 6\, m/s$

$\frac{dV}{dt} = ?$ when $s = 3$, $h = 9$

$\frac{dh}{dt} = -1\, m/s$

For MVT to apply on $[a,b]$,

- f must be continuous on $[a,b]$

$$f(c) = \lim_{x\to c} f(x) \quad \forall\, c \in (a,b)$$

"for all"

$$f(a) = \lim_{x\to a^+} f(x)$$

$$f(b) = \lim_{x\to b^-} f(x)$$

- f must be differentiable on (a,b)

$$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c} \quad \text{exists } \forall\, c \in (a,b)$$

$$= \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

1 Correct: 0%

This table gives a few values of function f.

x	0	1	2	3
$f(x)$	14	16	19	20

Omar said that since $\frac{f(3)-f(2)}{3-2} = 1$ there must be a number c in the interval $[2,3]$ for which $f'(c) = 1$.

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

Which condition makes Omar's claim true? → which guarantees that MVT applies?

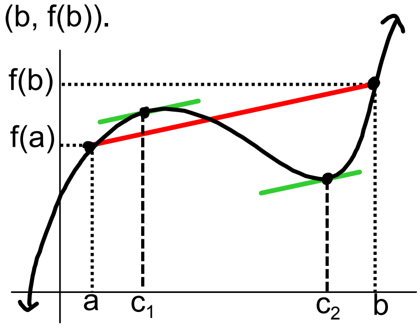
Choose 1 answer:

- (A) ~~f is increasing and continuous over the closed interval $[2,3]$.~~
- (B) f is continuous and differentiable over the open interval $(0,3)$. (ok if $[0,3]$)
- (C) ~~f is continuous over the open interval $(1,3)$.~~
- (D) f is differentiable over the closed interval $[2,3]$.

- continuity does not necessarily imply differentiability
- differentiability ALWAYS implies continuity

3.2 Rolle's Theorem & The Mean Value Theorem

The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).



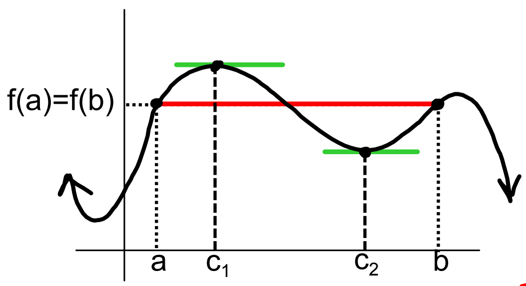
Is f(x) continuous on [a,b] & differentiable on (a,b)?
If so, then there exists $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

slope of tangent line @ (c, f(c))

slope of secant line through (a, f(a)) & (b, f(b))

Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



Criteria: f is continuous on [a,b] & differentiable on (a,b), $f(a)=f(b)$,

If so, Then, there exists $\exists\, c \in (a,b)$ s.t.

$$f'(c) = 0$$

Can Rolle's Theorem be applied?
If so, find all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4$, $[1,4]$

f is a polynomial, so cts on $[1,4]$ & diff on $(1,4)$

$f(1) \stackrel{?}{=} f(4)$ yes $\Rightarrow$ Rolle's Thm applies

$f(1) = 1 - 5 + 4 = 0$

$f(4) = 16 - 20 + 4 = 0$

$f'(x) = 2x - 5$

$2x - 5 = 0$

$x = 5/2$

Can the Mean Value Theorem be applied?
If so, find all guaranteed values of c in (a,b).

34. $f(x) = \dfrac{x+1}{x}$, $\left[\frac{1}{2}, 2\right]$

only problematic @ $x = 0$

<u>Steps to solve MVT problems</u>:

1. Is f continuous on [a,b]?
2. Is f differentiable on (a,b)?

} yes

3. Find (f(b)-f(a))/(b-a)
4. Find f'(x)
5. Set #3&4 equal, solve for x
6. Solution is the values of x from #5 that lie in (a,b)

$$\frac{f(b) - f(a)}{b - a} = \frac{\frac{2+1}{2} - \frac{\frac{1}{2}+1}{\frac{1}{2}}}{2 - \frac{1}{2}} = \frac{\frac{3}{2} - \left(\frac{1}{2}+1\right)(2)}{\frac{3}{2}} = \frac{\frac{3}{2} - (3)}{3/2} = \frac{-3}{\cancel{2}} \cdot \frac{\cancel{2}}{3} = -1$$

$$f'(x) = \frac{x(1) - (x+1)(1)}{x^2} = \frac{-1}{x^2}$$

$$(-x^2)\frac{-1}{x^2} = -1\,(-x^2)$$

$1 = x^2$

$\pm 1 = x$

$-1 \notin (1/2, 2)$

$\boxed{x = 1}$

38. $f(x) = 2\sin x + \sin 2x \, , \; [0, \pi]$

sinusoidal functions cts & diff on $\mathbb{R}$ ✓ MVT applies

$$\frac{f(\pi)-f(0)}{\pi - 0} = \frac{2\sin\pi + \sin 2\pi - (2\sin 0 + \sin 2(0))}{\pi - 0} = 0$$

$f'(x) = 2\cos x + 2\cos 2x$

$$2\cos x + 2\cos 2x = 0$$

$$2\cos x + 2(2\cos^2 x - 1) = 0$$

$$4\cos^2 x + 2\cos x - 2 = 0$$

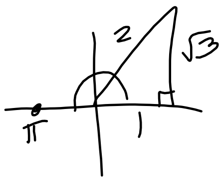
$$\frac{2(2\cos^2 x + \cos x - 1)}{2} = \frac{0}{2}$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -1$

$x = \pi/3$ $\qquad$ $\cancel{x = \pi \notin (0, \pi)}$



$f(x) = 2\sin x + \sin 2x$

$$\frac{d}{d\boxed{}}\boxed{}\;(2\sin(x) + \sin(2x))$$

x

solve$\left(2\cos(2x) + 2\cos(x) = 0, x\right)$