If f is continuous on $[a,b]$ and

$\lim_{x \to c} [f(x)] = f(c)$ $\forall x \in (a,b)$ & $\lim_{x \to a^+} f(x) = f(a)$, $\lim_{x \to b^-} f(x) = f(b)$

f is differentiable on (a,b),

$\lim_{x \to c} \frac{f(x)-f(c)}{x-c} = f'(c)$ $\forall x \in (a,b)$

then there exists at least one $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Note: Differentiability implies continuity, but continuity does not necessarily imply differentiability

This table gives a few values of function f.

x	7	8	9	10
$f(x)$	-11	-10	-11	-13

Leo said that since $\frac{f(8)-f(7)}{8-7} = 1$ there must be a number c in the interval $[7,8]$ for which $f'(c) = 1$.

Which condition makes Leo's claim true?

f cts. on $[7,8]$ & diff on $(7,8)$

Choose 1 answer:

(A) f is differentiable over the closed interval $[7,9]$. ✓

(B) f is continuous and differentiable over the open interval $(7,10)$. missing cont. @ 7

(C) $\lim_{x \to 7.5} f'(x) = 1$ weird, no

(D) f is differentiable over the open interval $(7,8)$ and continuous at $x = 8$.

32. $f(x) = x(x^2 - x - 2)$ $[-1, 1]$ MVT?

Is f cts. on $[-1,1]$? ✓ yes (polynomials are cts. on $\mathbb{R}$)
Is f diff on $(-1,1)$? ✓ yes (polynomials are diff. on $\mathbb{R}$)
$\Rightarrow$ Mean Value Theorem applies

$f(x) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$$\frac{f(1) - f(-1)}{1-(-1)} = \frac{1^3 - 1^2 - 2(1) - ((-1)^3 - (-1)^2 - 2(-1))}{2}$$

$$= \frac{\cancel{1} - \cancel{1} - \cancel{2} + 1 + 1 + \cancel{2}}{2}$$

$$= 1$$

$3x^2 - 2x - 2 = 1$

solve$(3x^2 - 2x - 2 = 1, x)$

$3x^2 - 2x - 3 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-3)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 + 36}}{6} = \frac{2 \pm \sqrt{40}}{6}$$

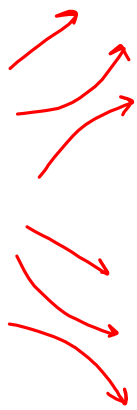
$$= \frac{2 \pm 2\sqrt{10}}{6} = \frac{1 \pm \sqrt{10}}{3}$$

$$= \cancel{\frac{1+\sqrt{10}}{3}}, \frac{1 - \sqrt{10}}{3}$$

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do f' and f'' tell us about f?

Recall that f' is the rate of change or slope of f,
f'' is the slope or rate of change of f'.

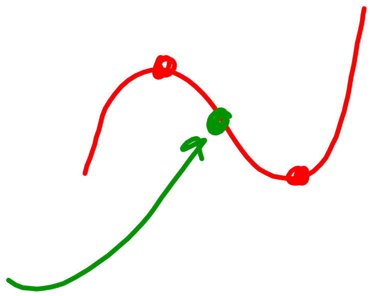


f'	f
$+$	↗ increasing
$-$	↘ decreasing

f''	f'	f
$+$	↗ increasing	concave up
$-$	↘ decreasing	concave down

f'(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undefined) are called <u>critical numbers</u>.

f''(x)=0 when f changes concavity.
The points where concavity changes are called <u>inflection points</u>.



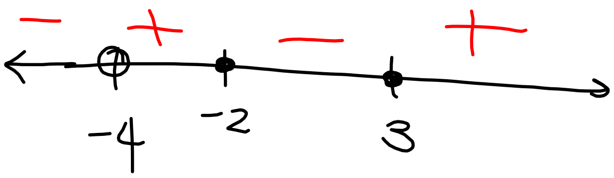
<u>3.1</u> #21,27,35 (Determine Absolute Extrema on a closed interval)

<u>3.2</u> #13,15,19 (Rolle's Theorem);
39,43,45 (Mean Value Theorem)

To solve problems involving concavity, increasing/decreasing, etc., we should recall how to solve polynomial inequalities.

$$\frac{(x+2)(x-3)}{x+4} \geq 0$$

$x = (-4, -2] \cup [3, \infty)$



- Find all critical numbers and state the open intervals on which f is increasing and/or decreasing.
- Find all inflection points and state the open intervals on which f is concave up and/or concave down.
- Use these results to determine all relative and absolute extrema.

3.3

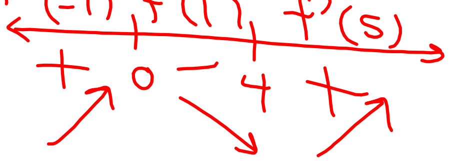
16. $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x$

$3x(x-4) = 0$

critical #'s : 0, 4

$f'(-1)$, $f'(1)$, $f'(5)$



f is increasing on $(-\infty, 0) \cup (4, \infty)$

f is decreasing on $(0, 4)$

f has a relative maximum @ $(0, 15)$

f has a relative minimum @ $(4, (4)^3 - 6(4)^2 + 15)$

$= (4, -17)$