$$d = 3h$$
$$2r = 3h$$
$$r = \frac{3h}{2}$$

$$V = \frac{1}{3}\pi\left(\frac{3h}{2}\right)^2 \cdot h$$
$$= \frac{\pi}{3} \cdot \frac{9h^2}{4} \cdot h$$
$$V = \frac{3\pi}{4} h^3$$

$$\frac{dh}{dt} = \frac{10}{81\pi}\ \text{ft/min}$$

3.4 #20

$$f(x) = \frac{x+1}{\sqrt{x}} = \frac{x+1}{x^{1/2}}$$

domain is $(0, \infty)$

$$f'(x) = \frac{x^{1/2}(1) - (x+1)\left(\frac{1}{2}x^{-1/2}\right)}{(x^{1/2})^2} = \frac{x^{1/2} - \frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}{x}$$
$$= \frac{\left(\frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}\right)}{x} \cdot \frac{2x^{1/2}}{2x^{1/2}} = \boxed{\frac{x-1}{2x^{3/2}} = f'(x) = 0 \ @\ x = 1}$$

$$f''(x) = \frac{(2x^{3/2})(1) - (x-1)(3x^{1/2})}{(2x^{3/2})^2} = \frac{2x^{3/2} - 3x^{3/2} + 3x^{1/2}}{4x^3} =$$
$$= \frac{3x^{1/2} - x^{3/2}}{4x^3} \cdot \frac{x^{1/2}}{x^{1/2}} = \frac{3x - x^2}{4x^{7/2}} = \frac{x(3-x)}{4x^{7/2}} = \boxed{\frac{3-x}{4x^{5/2}} \quad f''(x) = 0\ @\ x = 3}$$

f' = incr/decr: $f'(\frac{1}{2})$, $f'(2)$; 0 — 1 +

f'' = conc.: $f''(1)$, $f''(5)$; 0 + 3 —

f is increasing on $(1, \infty)$
& decreasing on $(0, 1)$
f has a relative minimum @ $(1, 2)$
f has no relative maxima

f is concave up on $(0, 3)$
& concave down on $(3, \infty)$
h has an inflection point @ $(3, \frac{4}{\sqrt{3}})$

3.7 Optimization Problems

4. Find two positive numbers (x & y) whose product is 192 and the sum of the first (x) plus three times the second (y) is a minimum.

sum	is	minimum

$$S(x,y) = x + 3y$$

$$xy = 192$$

$$x = \frac{192}{y}$$

$$S(y) = \frac{192}{y} + 3y$$

$$= 192y^{-1} + 3y$$

$$S'(y) = -192y^{-2} + 3$$

$$0 = -\frac{192}{y^2} + 3$$

$$\frac{192}{y^2} = 3$$

$$\frac{192}{3} = y^2$$

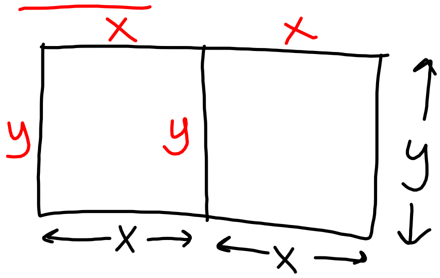
$$64 = y^2$$

$$8 = y$$

$$xy = 192$$

$$x = \frac{192}{8} = 24 = x$$

18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the figure. What dimensions should be used so that the enclosed area will be a maximum?



area	be (will?)	maximum

$$A(x,y) = (2x)\cdot y$$

$$A(y) = 2\left(50 - \tfrac{3}{4}y\right)\cdot y$$

$$= 100y - \tfrac{3}{2}y^2$$

$$A'(y) = 100 - 3y$$

$$100 - 3y = 0$$

$$100 = 3y$$

$$\frac{100}{3}\text{ ft} = y$$

$$200 = 4x + 3y$$

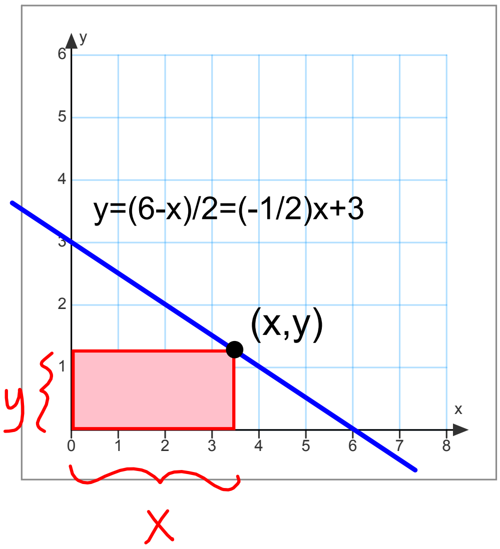
$$200 - 3y = 4x$$

$$50 - \tfrac{3}{4}y = x$$

$$x = 50 - \frac{\cancel{3}}{\cancel{4}} \cdot \frac{\cancel{100}^{25}}{\cancel{3}}$$

$$x = 25 \text{ ft}$$

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$$A(x,y) = xy$$

$$y = \frac{6-x}{2} = -\frac{1}{2}x + 3$$

$$A(x) = x\left(-\frac{1}{2}x + 3\right)$$

$$= -\frac{1}{2}x^2 + 3x$$

$$-x + 3 = 0$$

$$3 = x$$

$$3/2 = y$$

$$y = \frac{6-3}{2} = \frac{3}{2}$$