12. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}t$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time. Volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height.

$r = \sqrt{t+2}, \quad h = \frac{1}{2}t$

$V = \pi\left(\sqrt{t+2}\right)^2 \cdot \frac{1}{2}t$

$= \pi(t+2)\frac{t}{2}$

$V = \frac{\pi}{2}t^2 + \pi t$

$\frac{dV}{dt} = \pi t + \pi$

Locate the absolute extrema of the function on the closed interval. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$

$f'(x) = 3x^2 - 3x$

$3x(x-1) = 0$

$x = 0, 1$

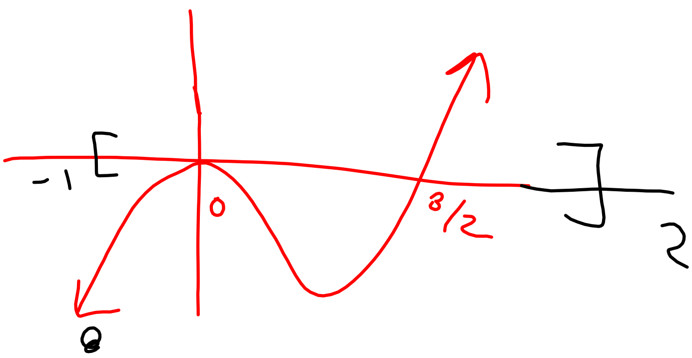
$f(-1) = -5/2 \leftarrow$ abs min

$f(0) = 0$

$f(1) = -1/2$

$f(2) = 2 \leftarrow$ abs max

$f(x) = x^2(x - 3/2)$



2. Determine if Rolle's Theorem can be applied to f on the closed interval [a , b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c)=0.

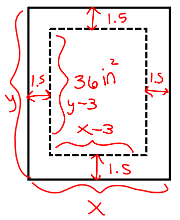
$$f(x) = (x-3)(x+1)^2\ , \qquad [-1, 3]$$

$f(-1) \overset{?}{=} f(3)$

f cts. on [a,b) & diff on (a,b)

3. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. $f(x) = x(x^2 - x - 2)\ ,\ [-1, 1]$

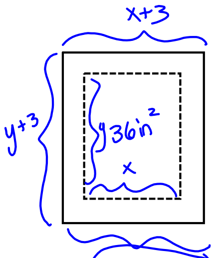
30. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the least amount of paper is used.



want to minimize area of page

$A(x,y) = xy$

$(x-3)(y-3) = 36$



$A(x,y) = (x+3)(y+3)$

$xy = 36$

$y = \frac{36}{x}$

$A(x) = (x+3)\left(\frac{36}{x}+3\right)$

$= 36 + 3x + \frac{3(36)}{x} + 9$

$A(x) = 45 + 3x + 3(36)x^{-1}$

$A'(x) = 3 - 3(36)x^{-2}$

$3 - \frac{3(36)}{x^2} = 0$

$3 = \frac{3(36)}{x^2}$

$3x^2 = 3(36)$

$x^2 = 36$

$x = 6$

$y = \frac{36}{x} = \frac{36}{6} = 6$

$\boxed{9'' \times 9''}$

7. Find the points of inflection and discuss concavity of the graph of the function. $f(x)=\frac{x}{x^2+1}$

$$f'(x)=\frac{(x^2+1)(1)-x(2x)}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}$$

$$f''(x)=\frac{(x^2+1)^2(-2x)-(1-x^2)(2(x^2+1)2x)}{(x^2+1)^4}$$

$$=\frac{(x^2+1)\left[-2x^3-2x-4x+4x^3\right]}{(x^2+1)^4}$$

$$=\frac{2x^3-6x}{(x^2+1)^3}=\frac{2x(x^2-3)}{(x^2+1)^3}=\frac{2x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}$$

$f''(x)=0$

when $2x=0$ or $x^2-3=0$

$x=0 \quad x=\pm\sqrt{3}$

$f''(-2)$, $f''(-1)$, $f''(1)$, $f''(2)$

$- \quad -\sqrt{3} \quad + \quad 0 \quad - \quad \sqrt{3} \quad +$

f is concave down on $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$ & up on $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$

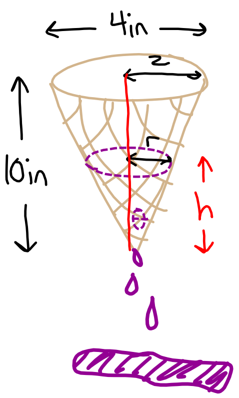
inflection pts @ $(-\sqrt{3},f(-\sqrt{3})),(0,f(0)),(\sqrt{3},f(\sqrt{3}))$

f''

$- \quad a \quad - \quad b \quad + \quad c \quad -$

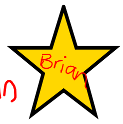
no inflection point @ a

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$\frac{dV}{dt}=?\ \text{in}^3/\text{min}$ when $h=5\text{in}$

$\frac{dh}{dt}=\frac{-1\text{in}}{5\text{ min}}$



$V=\frac{\pi}{3}r^2h$

$\frac{r}{h}=\frac{2}{10}\Rightarrow r=\frac{h}{5}$

$V=\frac{\pi}{3}\left(\frac{h}{5}\right)^2h$

$V=\frac{\pi}{75}h^3$

$\frac{dV}{dt}=\frac{\pi}{25}h^2\cdot\frac{dh}{dt}$

$=\frac{\pi}{25}(5)^2\cdot\frac{-1}{5}$

$=\frac{-\pi}{5}\ \text{in}^3/\text{min}$