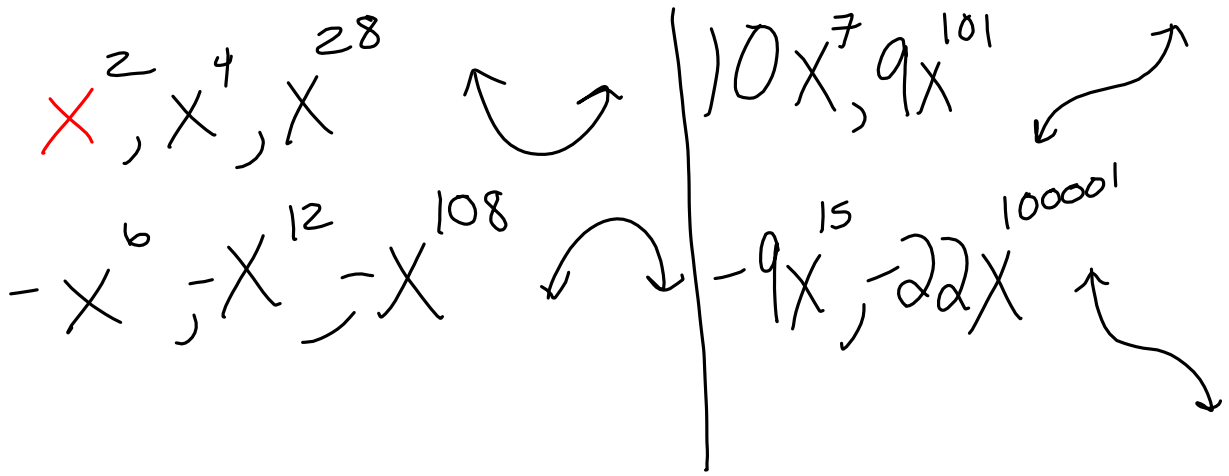
3.5 Limits at Infinity

$\lim_{x \to \infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes



$$f(x) = \frac{5x^2-3x+4}{2x^2+5x} \approx \frac{5x^2}{2x^2} = \frac{5}{2}$$

Horizontal asymptote @ $y = \frac{5}{2}$

$$\lim_{x\to\infty} f(x) = \frac{5}{2} = \lim_{x\to-\infty} f(x)$$

$$f(x) = \frac{2x-4}{3x^4} \approx \frac{2x}{3x^4} = \frac{2}{3x^3} \to 0$$

Horizontal asymptote @ $y = 0$

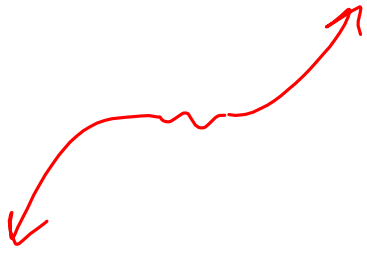
$$\lim_{x\to-\infty} f(x) = 0 = \lim_{x\to\infty} f(x)$$

$$f(x) = \frac{2x^7-4x^3-2}{5x^4+1} \approx \frac{2x^7}{5x^4} = \frac{2}{5}x^3$$

positive, odd

$$\lim_{x\to-\infty} f(x) = -\infty$$

$$\lim_{x\to\infty} f(x) = \infty$$



$$f(x) = \frac{2-7x^3+2x}{1+x}$$

24. $\lim_{x \to -\infty} \left(\frac{1}{2}x - \frac{4}{x^2} \right) = -\infty - 0$

$= -\infty$

$\frac{x^2}{x^2} \cdot \frac{x}{2} - \frac{4}{x^2} \cdot \frac{2}{2}$

$= \lim_{x \to -\infty} \frac{x^3 - 8}{2x^2} = \lim_{x \to -\infty} \frac{x^3}{2x^2} = \lim_{x \to -\infty} \frac{1}{2}x$

$= -\infty$

$$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$$

$\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$

$\sqrt[2]{(-2)^2} = \sqrt{4} = 2 = |-2|$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|2| = 2$$

$$|-2| = 2 = -(-2)$$

26. $\lim_{x \to -\infty} \frac{x}{\sqrt{x^2+1}} \approx \lim_{x \to -\infty} \frac{x}{\sqrt{x^2}}$

$$= \lim_{x \to -\infty} \frac{x}{|x|} = \lim_{x \to -\infty} \frac{x}{-x}$$

$$= \lim_{x \to -\infty} (-1) = \boxed{-1}$$

$$\lim_{x\to\infty} \frac{5x-2}{\sqrt{9x^2+3}} = \lim_{x\to\infty} \frac{5x}{\sqrt{9x^2}} = \lim_{x\to\infty} \frac{5x}{|3x|}$$

$$= \lim_{x\to\infty} \frac{5x}{3x} = \lim_{x\to\infty} \frac{5}{3} = \boxed{\frac{5}{3}}$$

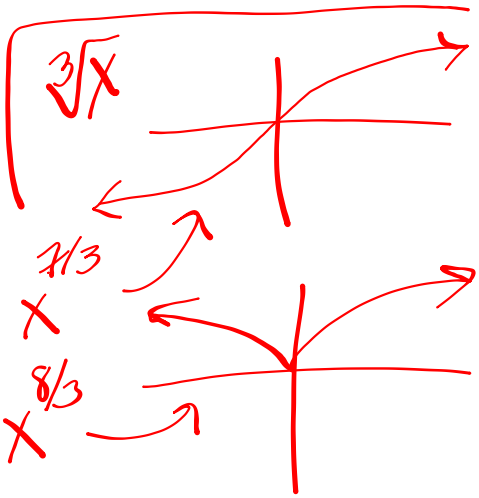
$$\lim_{x\to-\infty} \frac{9x^3-4x+3}{\sqrt[3]{-27x^2-40000000x+7}}$$

$$= \lim_{x\to-\infty} \frac{9x^3}{\sqrt[3]{-27x^2}} = \lim_{x\to-\infty} \frac{9x^3}{-3x^{2/3}}$$

$$= \lim_{x\to-\infty} -3x^{7/3}$$

$$= (-3)(-\infty)$$

$$= \boxed{\infty}$$



30. $\lim_{x\to\infty} \dfrac{x-\cos x}{x} = \lim_{x\to\infty}\left(\dfrac{x}{x} - \dfrac{\cos x}{x}\right)$

$= \lim_{x\to\infty}\left(1 - \dfrac{\cos x}{x}\right) = \lim_{x\to\infty}(1) - \lim_{x\to\infty}\left(\dfrac{\cos x}{x}\right)$

$= 1 - \lim_{x\to\infty} \dfrac{\cos x}{x}$ ← bounded by $[-1,1]$

$= 1 - \dfrac{\text{small}}{\infty} = 1 - 0 = \boxed{1}$

$\lim_{x\to\infty} \dfrac{x^3 - x^4 - x^2}{2 - x^3 + 3x} = \lim_{x\to\infty} \dfrac{\frac{x^3}{x^3} - \frac{x^4}{x^3} - \frac{x^2}{x^3}}{\frac{2}{x^3} - \frac{x^3}{x^3} + \frac{3x}{x^3}}$ (the $\frac{x^2}{x^3}$, $\frac{2}{x^3}$, and $\frac{3x}{x^3}$ terms $\to 0$)

$= \lim_{x\to\infty} \dfrac{1-x}{1} = 1 - (\infty) = \boxed{-\infty}$

$$32.\ \lim_{x\to\infty} \cos\frac{1}{x} = \cos 0 = \boxed{1}$$