$$x + 2y = 56 \longrightarrow x = 56 - 2y$$

$$P(x,y) = xy$$

$$P(y) = (56-2y)y = 56y - 2y^2$$

$$P'(y) = 56 - 4y$$

$$56 - 4y = 0$$

$$56 = 4y$$

$$14 = y \Rightarrow x = 56 - 2y = 56 - 2(14)$$

$$x = 28$$

Find the limit as x approaches *negative* infinity.

$$\lim_{x \to -\infty} \frac{\sqrt{4x^4 - x}}{2x^2 + 3} = \lim_{x \to -\infty} \frac{\sqrt{4x^4}}{2x^2} = \lim_{x \to -\infty} \frac{|2x^2|}{2x^2}$$

$$= \lim_{x \to -\infty} \frac{2x^2}{2x^2} = \lim_{x \to -\infty} 1 = \boxed{1}$$

Find the limit as x approaches *negative* infinity.

$$\lim_{x\to-\infty} \frac{x^5+4x^2}{\sqrt{x^{10}+8x^7}} = \lim_{x\to-\infty} \frac{x^5}{\sqrt{x^{10}}} = \lim_{x\to-\infty} \frac{x^5}{|x^5|}$$

$$= \lim_{x\to-\infty} \frac{x^5}{-x^5} = \lim_{x\to-\infty} (-1) = \boxed{-1}$$

$$\lim_{x\to\infty} \frac{2x^2-x^4+3x-7x^5}{\sqrt[3]{2+x-x^6}}$$

$$= \lim_{x\to\infty} \frac{-7x^5}{\sqrt[3]{-x^6}} = \lim_{x\to\infty} \frac{-7x^5}{-x^2}$$

$$= \lim_{x\to\infty} 7x^3 = \boxed{\infty}$$

$$\sqrt[3]{-x^6} = \sqrt[3]{(-1)^3(x^2)^3} = -x^2$$

18. c.

$$\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1} = \lim_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}}$$

$$= \lim_{x\to\infty} \frac{5}{4}x = \boxed{\infty}$$

7.7 Indeterminate Forms & L'Hôpital's Rule

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty$, 1^{∞}, 0^0, *and* $\infty-\infty$ are called <u>indeterminate forms</u>.

$\frac{-\infty}{-\infty}$, $-\frac{\infty}{\infty}$

<u>L'Hôpital's Rule</u>:

Let f and g be functions that are differentiable on an open interval (a,b) containing c, except possibly at c itself. Assume that $g'(x)\neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0$, ∞/∞, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

7.7

12. $$\lim_{x\to -1} \frac{x^2-x-2}{x+1} = \lim_{x\to -1} \frac{(x-2)\cancel{(x+1)}}{\cancel{x+1}} = \boxed{-3}$$

$= \frac{0}{0}$ l'hopital applies

$$= \lim_{x\to -1} \frac{2x-1}{1} = 2(-1)-1 = \boxed{-3}$$

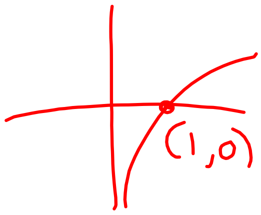
16. $$\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3} = \frac{e^0-(1+0)}{0^3} = \frac{0}{0}$$

l'Hopital applies

$$= \lim_{x\to 0^+} \frac{e^x-1}{3x^2} = \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{e^x}{6x} = \frac{1}{0^+} = \boxed{\infty}$$

18. $\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1} = \frac{\ln 1}{1 - 1} = \frac{0}{0}$



$= \lim_{x \to 1} \frac{\frac{1}{x^2} \cdot \cancel{2x}}{\cancel{2x}} = \frac{1}{1^2} = \boxed{1}$

20. $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{0}{0}$

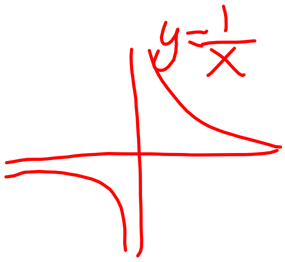
$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$= \lim_{x \to 0} \frac{a \cos ax}{b \cos bx} = \frac{a \cdot 1}{b \cdot 1} = \boxed{\frac{a}{b}}$

28. $\lim_{x\to\infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2x}{e^x} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = \boxed{0}$



$\frac{\pm C}{\pm\infty} \to 0$

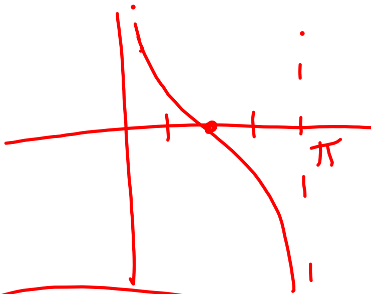
$\frac{+C}{0^+} \to \infty$ $\qquad$ $\frac{-C}{0^+} \to -\infty$

$\frac{+C}{0^-} \to -\infty$ $\qquad$ $\frac{-C}{0^-} \to +\infty$

36. $$\lim_{x\to\infty} \frac{e^{x/2}}{x} = \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{2}e^{x/2}}{1} = \boxed{\infty}$$

38. $$\lim_{x\to 0^+} x^3 \cot x = 0\cdot\infty$$



$$= \lim_{x\to 0^+} \frac{x^3}{\left(\frac{1}{\cot x}\right)} = \lim_{x\to 0^+} \frac{\cot x}{\left(\frac{1}{x^3}\right)}$$

$$0\cdot\infty = 0\cdot\frac{1}{0}$$

$$\frac{1}{\infty}\cdot\infty$$

$$= \lim_{x\to 0^+} \frac{x^3}{\tan x} = \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{3x^2}{\sec^2 x} = \frac{3\cdot 0}{1} = \boxed{0}$$

40. $\lim\limits_{x\to\infty} x\tan\frac{1}{x} = \infty \cdot 0$

$= \lim\limits_{x\to\infty} \dfrac{\tan\frac{1}{x}}{\frac{1}{x}} = \dfrac{0}{0}$

$= \lim\limits_{x\to\infty} \dfrac{\left(\sec^2\frac{1}{x}\right)\cdot -\cancel{\frac{1}{x^2}}}{-\cancel{\frac{1}{x^2}}} = \boxed{1}$