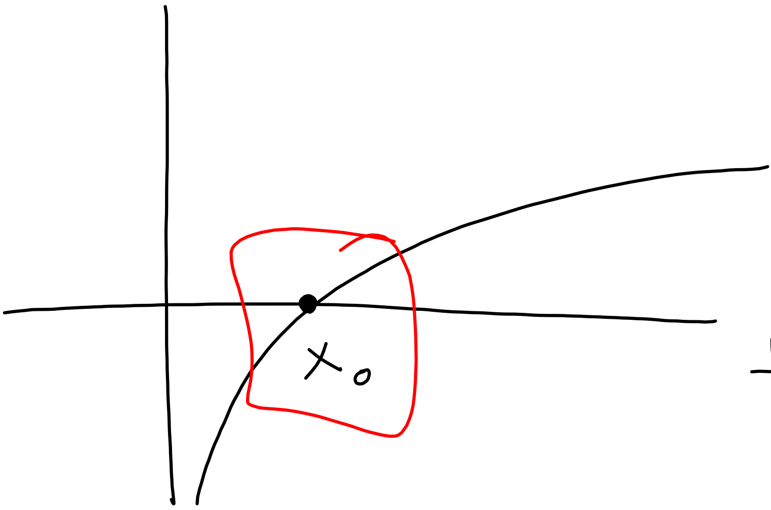
42. $\lim_{x \to 0^+} (e^x + x)^{2/x}$

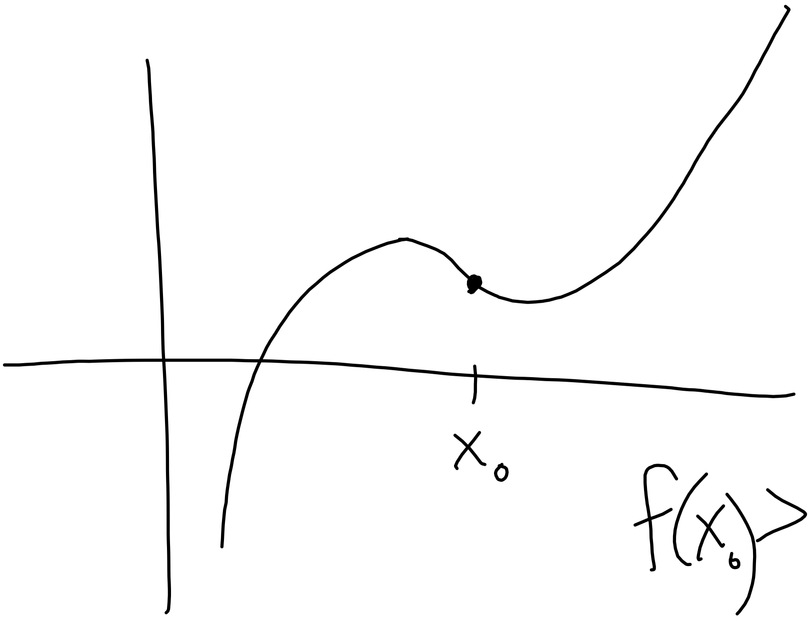


$f(x_0) = 0$

$f''(x_0) < 0$

$f'(x_0) > 0$

$f'(x_0) > f(x_0) > f''(x_0)$



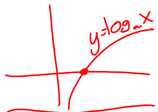
$$f(x_0) > 0$$
$$f'(x_0) < 0$$
$$f''(x_0) = 0$$
$$f(x_0) > f''(x_0) > f'(x_0)$$

44. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = 1^\infty \neq 1$

$$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$
$$\ln y = \ln\left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right)$$
$$\ln y = \lim_{x\to\infty}\left[\ln\left(1+\frac{1}{x}\right)^x\right]$$
$$\ln y = \lim_{x\to\infty}\left[x\ln\left(1+\frac{1}{x}\right)\right]$$
$$\infty \cdot 0$$
$$\ln y = \lim_{x\to\infty}\left[\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}\right] = \frac{0}{0}$$

l'hopital's rule applies

$$\ln y = \lim_{x\to\infty}\left[\frac{\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{\cancel{x^2}}}{\frac{-1}{\cancel{x^2}}}\right]$$
$$\ln y = 1$$
$$e^{\ln y} = e^1$$
$$y = \boxed{e}$$



$$\lim_{x\to c}\ln\left(x^2\right) = \ln\left(\lim_{x\to c}\left(x^2\right)\right)$$

$$\log_a\left(x^p\right) = p\log_a x$$

$$a^{\log_a x} = x$$
$$\log_a a^x = x$$

$\lim_{x\to 0^+} (\sin x)^x = 0^0$ (indeterminate form)

step 1 - try to plug in limit $(\sin 0)^0 = 0^0$

step 2 - set equal to y $y = \lim_{x\to 0^+} (\sin x)^x$

step 3 - take log of both sides $\ln y = \ln\left[\lim_{x\to 0^+} (\sin x)^x\right]$

step 4 - interchange limit & log $\ln y = \lim_{x\to 0^+}\left[\ln (\sin x)^x\right]$

step 5 - apply power rule for logs $\ln y = \lim_{x\to 0^+}\left[x \cdot \ln(\sin x)\right]$

step 6 - try again to plug in limit $0 \cdot \ln(\sin 0) = 0 \cdot (-\infty)$ $\ln(0^+)$

step 7 - rewrite as $\frac{0}{0}$ or $\frac{\infty}{\infty}$ $\ln y = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}}$

step 8 - apply l'hopital's rule $\ln y = \lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot \cos x}{-\frac{1}{x^2}}$

step 9 - rewrite before applying l'h again $\ln y = \lim_{x\to 0^+} \frac{-x^2}{\tan x} = \frac{0}{0}$

step 10 - l'hopital $\ln y = \lim_{x\to 0^+} \frac{-2x}{\sec^2 x} = \frac{0}{1}$

step 11 - plug in limit $\ln y = 0$

step 12 - apply exponential function $e^{\ln y} = e^0$ $e^{\log_e y}$

$y = \boxed{1}$

3. Find $f'(3)$ given that $g(3) = 2,\ g'(3) = -1,\ h(3) = 5,$ and $h'(3) = -2$

a. $f(x) = 2g(x)h(x)$

b. $f(x) = 2g(x) + h(x)$

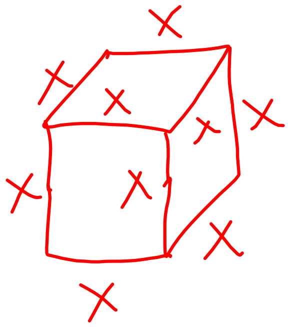
c. $f(x) = 7 + \frac{g(x)}{h(x)}$

$$f'(x) = 0 + \frac{h(x)\cdot g'(x) - g(x)\cdot h'(x)}{[h(x)]^2}$$

$$f'(3) = \frac{h(3)\cdot g'(3) - g(3)h'(3)}{[h(3)]^2} = \frac{5(-1) - 2(-2)}{(5)^2} = -\frac{1}{25}$$

10. All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is 10 centimeters?

$\frac{dx}{dt} = 3 \text{ cm/s}$; $\frac{dV}{dt} = ?$ when $x = 10\text{cm}$



$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} = 3(10)^2 \cdot (3)$$

$$= \boxed{900 \text{ cm}^3/\text{s}}$$