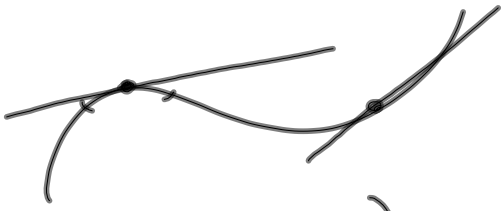
3.9 Differentials



$y - y_1 = m(x - x_1)$; $(c, f(c))$

$y - f(c) = f'(c)(x - c)$

$\boxed{y = f(c) + f'(c)(x - c)}$

Equation of Tangent line to f @ c.

2. $f(x) = \frac{6}{x^2}$; $(2, \frac{3}{2})$

compare actual function values w/ tangent line approximation near 2.

tangent: $y = f(c) + f'(c)(x - c)$

$f(x) = 6x^{-2}$

$f'(x) = -12x^{-3} = \frac{-12}{x^3}$

$f'(2) = \frac{-12}{2^3} = \frac{-12}{8} = \frac{-3}{2}$

$y = \frac{3}{2} + \left(\frac{-3}{2}\right)(x - 2)$

$= \frac{3}{2} - \frac{3}{2}x + 3$

$\boxed{y = \frac{9}{2} - \frac{3}{2}x}$ Tangent line $T(x)$ v. $f(x) = \frac{6}{x^2}$

x	1.9	1.99	2	2.01	2.1
f(x)	1.66	1.51	1.5	1.49	1.36
T(x)	1.65	1.515	1.5	1.485	1.35

$$y = f(c) + f'(c)(x-c)$$

$$y - f(c) = f'(c)(x-c)$$

$$= f'(c)\,\Delta x \longleftarrow dx$$

$$\Delta y = f(x+c) - f(x)$$

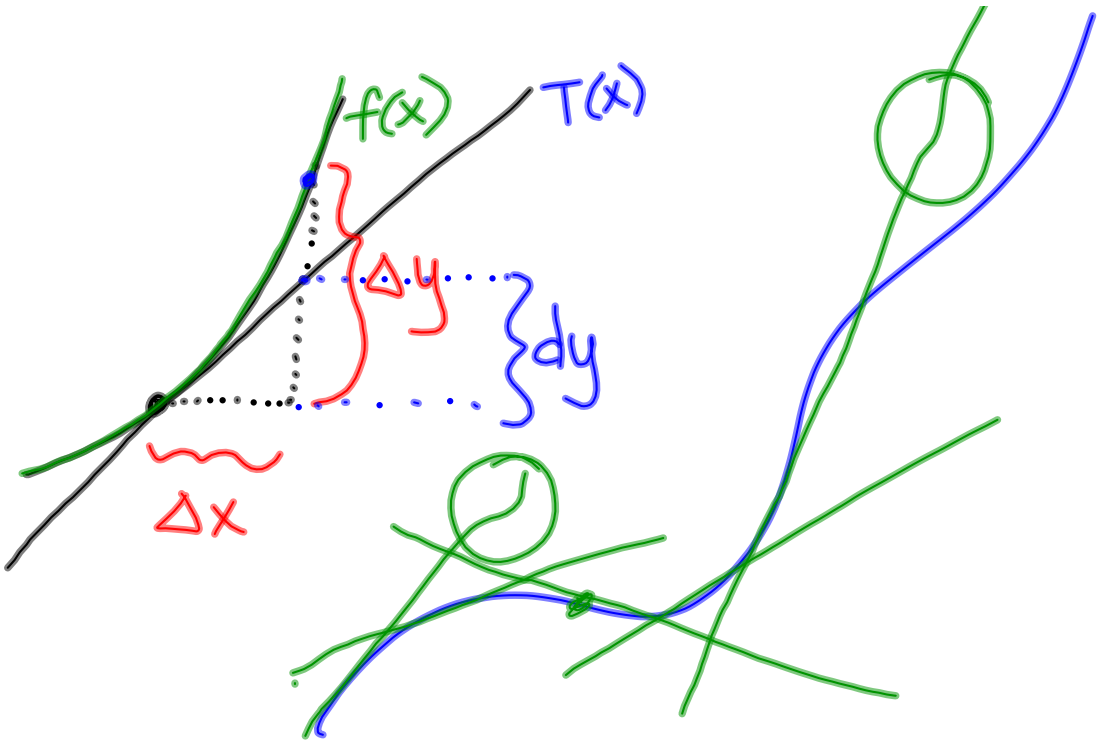
$$\Delta y \approx f'(c)\,dx$$

$$dy = f'(c)\,dx$$

The differential of y is

$$dy = f'(x)dx$$

$$\frac{dy}{dx} = f'(x)$$



8. $y = 1-2x^2 = f(x)$; $x=0$; $\Delta x = dx = -0.1$

compare dy & Δy

$f(c+\Delta x) - f(c)$

$\Delta y = f(0+(-0.1)) - f(0)$

$= 1-2(0+(-0.1))^2 - [1-2(0)^2]$

$= 1-0.02-1 = -0.02$

$dy = f'(c)\cdot dx$ $\quad f'(x) = -4x$

$= -4(0)\cdot(-0.1)$

$= 0$

Find the differential dy.

$dy = f'(x)dx$

12. $y = 3x^{2/3}$

$dy = 2x^{-1/3}dx$

14. $y = \sqrt{9-x^2}$

$dy = \frac{1}{2}(9-x^2)^{-1/2}(-2x)dx$

16. $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$dy = \left(\frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}\right)dx$

20. $y = \frac{\sec^2 x}{x^2+1}$

$dy = \frac{(x^2+1)(2\sec^2 x\tan x) - \sec^2 x(2x)}{(x^2+1)^2}dx$

46. $\sqrt[3]{26}$

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3(\sqrt[3]{x})^2}$

$f(x) = \sqrt[3]{x}$; $x = 27$; $\Delta x = dx = -1$

$\Delta y \approx f'(x) \cdot \Delta x$

$f(x+\Delta x) - f(x) \approx f'(x)(\Delta x)$

$f(x+\Delta x) \approx f(x) + f'(x)\,dx$

$\sqrt[3]{26} = \sqrt[3]{27-1} \approx \sqrt[3]{27} + \left(\frac{1}{3(\sqrt[3]{27})^2}\right)(-1)$

$\approx 3 - \frac{1}{27} \approx 2.96296$

$x^{m/n} = (x^m)^{1/n} = (x^{1/n})^m$

$= \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

50. $\tan(0.05)$

$x = 0 \quad ; \quad \Delta x = 0.05$

$f(x+\Delta x) = f(x) + f'(x)\Delta x$

$\tan(0.05) = \tan 0 + \sec^2(0)(0.05)$

$= 0 + 1 \cdot 0.05$

$\approx \boxed{0.05}$

HW

3.9 #5, 9, 11-19 odd

45, 49