

Review:

approximate $\cos(0.02)$

$$f(x) = \cos x$$

$$x = 0; dx = 0.02$$

$$f(x + \Delta x) = f(x) + f'(x)(dx)$$

$$\cos(0 + 0.02) = \cos 0 + (-\sin 0)(0.02)$$

$$= \boxed{1}$$

4.1 Antiderivatives

$$F(x) = 5x^4$$

$$f(x) = x^5$$

$$[f(x)]' = 5x^4$$

↑
particular solution

General solution: $x^5 + C$

$$56. \quad g'(x) = 6x^2, \quad g(0) = -1$$

$$y' = 6x^2$$

$$g(x) = 2x^3 + C \quad \leftarrow \text{general solution}$$

$$-1 = 0 + C$$

$$-1 = C$$

particular solution:

$$g(x) = 2x^3 - 1$$

$$y = F(x)$$

$$\frac{dy}{dx} = f(x)$$

antiderivative
=

$$\int dy = \int f(x) dx$$

indefinite
integral

$$y = \int f(x) dx = F(x) + C$$

$$18. \int (4x^3 + 6x^2 - 1) dx$$

$$= x^4 + 2x^3 - x + C$$

$$24. \int (\sqrt[4]{x^3} + 1) dx = \int (x^{3/4} + 1) dx$$

$$\frac{x^{7/4}}{7/4} + x + C = \boxed{\frac{4}{7} x^{7/4} + x + C}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$x^{-1} = \frac{1}{x}$$

$$28. \int \frac{x^2 + 2x - 3}{x^4} dx = \int \left(\frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{3}{x^4} \right) dx$$

$$= \int (x^{-2} + 2x^{-3} - 3x^{-4}) dx$$

$$= \boxed{-x^{-1} - x^{-2} + x^{-3} + C}$$

$$38. \int (\theta^2 + \sec^2 \theta) d\theta$$

$$= \boxed{\frac{\theta^3}{3} + \tan \theta + C}$$

$$42. \int \frac{\cos x}{1 - \cos^2 x} dx = \int \frac{\cos x}{\sin^2 x} dx$$

$$= \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} dx$$

$$= \int \csc x \cot x dx = \boxed{-\csc x + C}$$

$$40. \int \sec y (\tan y - \sec y) dy$$

$$= \int (\sec y \tan y - \sec^2 y) dy$$

$$= \boxed{\sec y - \tan y + C}$$

$$58. f'(s) = 6s - 8s^3, f(2) = 3$$

$$f(s) = 3s^2 - 2s^4 + C \quad \text{general solution}$$

$$3 = 3(2)^2 - 2(2)^4 + C$$

$$3 = 12 - 32 + C$$

$$23 = C$$

particular solution: $f(s) = 3s^2 - 2s^4 + 23$

$$62. f''(x) = \sin x, f'(0) = 1, f(0) = 6$$

$$f'(x) = -\cos x + C_1$$

$$f(x) = -\sin x + C_1 x + C_2$$

general solution

$$1 = -\cos(0) + C_1, \quad 6 = -\sin(0) + C_1(0) + C_2$$

$$C_1 = 2$$

;

$$C_2 = 6$$

particular solution: $f(x) = -\sin x + 2x + 6$

$s(t) = \text{position}$ s m

$v(t) = s'(t) = \text{velocity}$ $\frac{\Delta s}{\Delta t}$ m/s

$a(t) = v'(t) = s''(t) = \text{acceleration}$
 $\frac{\frac{\Delta s}{\Delta t}}{\Delta t}$ m/s^2

$$72. \quad 1600 \text{ m}$$

$$a = -9.8 \text{ m/s}^2$$

$$a(t) = -9.8$$

$$v(t) = -9.8t + v_0$$

$$s(t) = -\frac{9.8}{2} t^2 + v_0 t + s_0$$

$$s(t) = -\frac{9.8}{2} t^2 + 1600$$

$$0 = -\frac{9.8}{2} t^2 + 1600$$

$$\sqrt{\frac{3200}{9.8}} \text{ s} = t$$

80. $a(t) = \cos t$, $t > 0$
 @ $t=0$; position is $x=3$ $s(0)=3$
 $v(0)=0$

a) find velocity & position functions

b) find value(s) of t for which the particle is at rest.

$$v(t) = \int \cos t \, dt = \sin t + C_1$$

$$0 = \sin(0) + C_1$$

$$C_1 = 0$$

$$v(t) = \sin t$$

$$s(t) = \int \sin t \, dt = -\cos t + C_2$$

$$3 = -\cos(0) + C_2$$

$$v(t) = 0$$

$$-\sin t = 0$$

$$\sin t = 0$$

$$t = 0, \pi, 2\pi, 3\pi, \dots$$

$$4 = C_2$$

$$s(t) = -\cos t + 4$$

$$t = k\pi, \quad k \geq 0 \text{ integer}$$

4.1 HW

5-33 odd

55-61 odd

67, 83