3.9 Differentials

The tangent line to the graph of f at the point $(c, f(c))$, according to the point-slope equation is $y - f(c) = f'(c)(x - c)$ or, rearranged, $y = f(c) + f'(c)(x - c)$, where we call the change in x, $x - c = \Delta x$.

Actual change in y is given by $\Delta y = f(c + \Delta x) - f(c)$. When Δx is small, change in y can be approximated by $\Delta y \approx f'(c)\Delta x$.

For such an approximation, Δx is denoted dx, and is called the differential of x.

For a differentiable function $y = f(x)$, the differential of y is $\boxed{dy = f'(x)dx}$

The approximate function value at $c + \Delta x$ can be found by $\boxed{f(c + \Delta x) = f(c) + f'(c)\Delta x}$

4.1 Antiderivatives

$F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$ for all x on a given interval.

A synonym for antiderivative is indefinite integral, and if $F'(x) = f(x)$, we write $\int f(x)dx = F(x) + C$.

$$(\cot x)' = -\csc^2 x$$

4.1 how high will ball go?

67. $s_0 = 6\text{ ft}$, $v_0 = 60\text{ ft/s}$; $a(t) = -32\text{ ft/s}^2$

$a(t) = -32$

$v(t) = -32t + C$

$v(0) = -32(0) + C$

$60 = C$

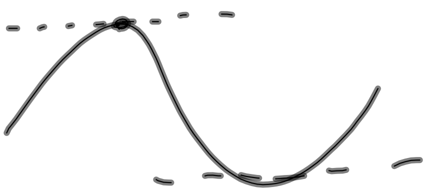
$v(t) = -32t + 60$

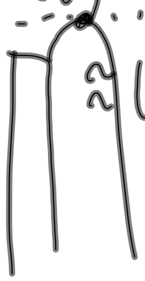
$s(t) = -16t^2 + 60t + 6$

$v(t) = 0$

$t = \frac{60}{32} = \frac{30}{16} = \frac{15}{8}$

$s\left(\frac{15}{8}\right) = \text{max height}$





$\approx 62\text{ ft}$

4.1

car

83. $a(t) = 6$ ft/s^2

$v_0 = 0$

truck

$v(t) = 30$ ft/s

$s(t) = 30t + s_0$

$v(t) = 6t + v_0$

$s(t) = 3t^2 + s_0$

$3t^2 = 30t$

a) 300 ft

b) 60 ft/s

$3t(t-10) = 0$

$t = 0 \quad t = 10$

27. $$\int \frac{x^2 + x + 1}{\sqrt{x}}\,dx = \int \left(\frac{x^2}{x^{1/2}} + \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}}\right)dx$$

$$= \int \left(x^{2-\frac{1}{2}} + x^{1-\frac{1}{2}} + x^{-1/2}\right)dx$$

$$= \int \left(x^{3/2} + x^{1/2} + x^{-1/2}\right)dx$$

$$= \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

3.9

9. $y=x^4+1$ compare Δy & dy

$dy = 4x^3 dx$

$\Delta y = f(x+\Delta x) - f(x) = (x+\Delta x)^4 + 1 - (x^4+1)$

$x=-1,$
$\Delta x = dx = 0.01$

$= (x+\Delta x)^4 - x^4$

$$f(c+\Delta x) = f(c) + f'(c)\Delta x$$

$\sin 3$; $f(x) = \sin x$

$\approx 0 - 1(\pi - 3)$

$c = \pi$

$f(c) = \sin\pi = 0$

$f'(x) = \cos x$

$f'(\pi) = \cos\pi = -1$

4.2 Area

Sigma Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

Summation Formulas

1. $\sum_{i=1}^{n} c = nc$

2. $\sum_{i=1}^{n} i = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\ldots+n^3 = \frac{n^2(n+1)^2}{4}$

8. $\frac{5}{1+1}+\frac{5}{1+2}+\frac{5}{1+3}+\dots+\frac{5}{1+15}$

$$\sum_{i=1}^{15}\frac{5}{1+i}$$

14. $\left(\frac{1}{n}\right)\sqrt{1-\left(\frac{0}{n}\right)^2}+\dots+\left(\frac{1}{n}\right)\sqrt{1-\left(\frac{n-1}{n}\right)^2}$

$$\sum_{i=1}^{n}\frac{1}{n}\sqrt{1-\left(\frac{i-1}{n}\right)^2}$$

20. $\sum_{i=1}^{10} i(i^2+1)=\sum_{i=1}^{10}(i^3+i)$

$$=\sum_{i=1}^{10} i^3+\sum_{i=1}^{10} i$$

$$=\frac{10^2(10+1)^2}{4}+\frac{10(10+1)}{2}$$

$$=25(121)+55=\boxed{3080}$$

4.2

#7-19 odd