1. $y = 3x^2 - 4$

$dy = f'(x)\,dx$

$dy = \boxed{6x\,dx}$

2. $y = 2x - \cot^2 x$

$y = 2x - [\cot x]^2$

$dy = [2 - 2\cot x \cdot (-\csc^2 x)]$

$dy = \boxed{[2 + 2\cot x \csc^2 x]\,dx}$

3. $\int (x^3 + 2)\,dx = \boxed{\frac{x^4}{4} + 2x + c}$

4. $\int \left(\sec^2\theta - \sin\theta\right) d\theta$

$= \boxed{\tan\theta + \cos\theta + C}$

$$f(c+\Delta x) \approx f(c) + f'(c)\Delta x$$

5. $\sqrt{99}$

$$f(x) = \sqrt{x} \quad ; \; c = 100$$

$$\Delta x = -1 \quad ; \; f'(x) = \frac{1}{2\sqrt{x}}$$

$$\sqrt{100-1}$$

$$\approx \sqrt{100} + \frac{1}{2\sqrt{100}}(-1)$$

$$= 10 - \frac{1}{20}$$

$$= \frac{199}{20} = 9.95$$

4.2

11. $$\left[\left(\frac{2}{n}\right)^3 - \frac{2}{n}\right]\left(\frac{2}{n}\right) + \ldots + \left[\left(\frac{2n}{n}\right)^3 - \frac{2n}{n}\right]\left(\frac{2}{n}\right)$$

$$\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^3 - \frac{2i}{n}\right]\left(\frac{2}{n}\right)$$

15. $\sum_{i=1}^{20} 2i = 2\sum_{i=1}^{20} i$

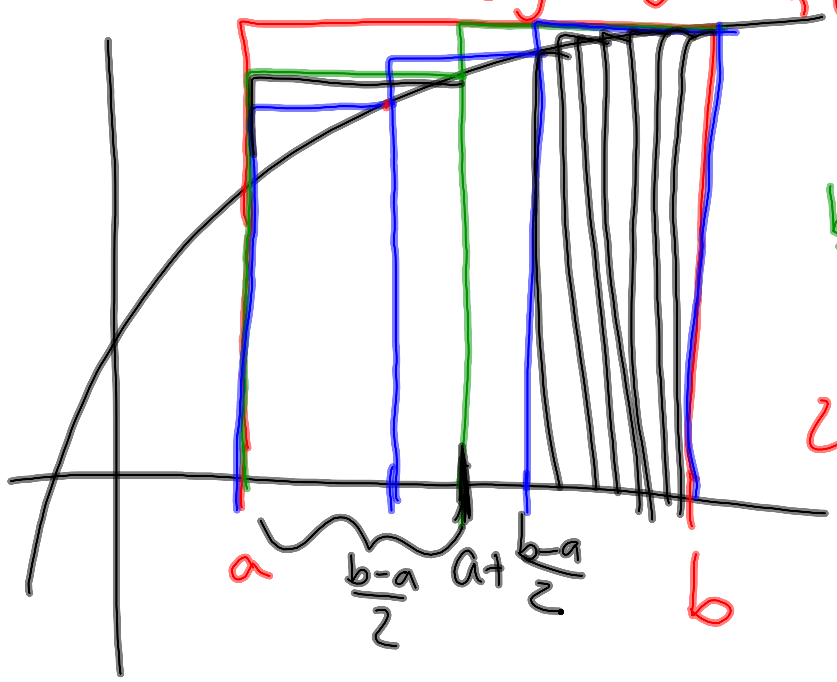
19. $\sum_{i=1}^{15} i(i-1)^2 = \sum_{i=1}^{15}\left(i^3-2i^2+i\right)$

$i(i^2-2i+1)$

$=\sum_{i=1}^{15} i^3 - 2\sum_{i=1}^{15} i^2 + \sum_{i=1}^{15} i$

4.2 Area

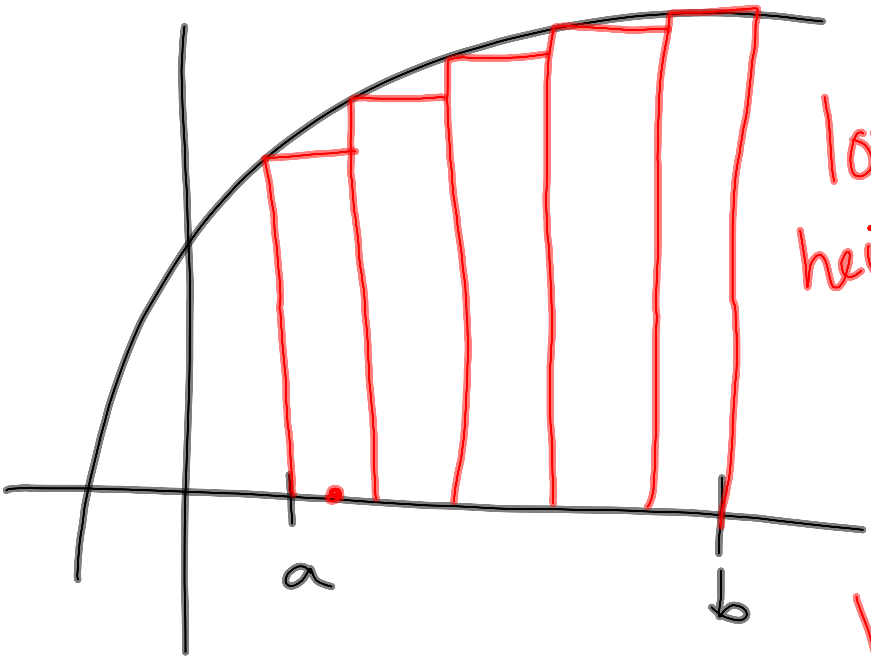
height of □ is det. by right endpoint



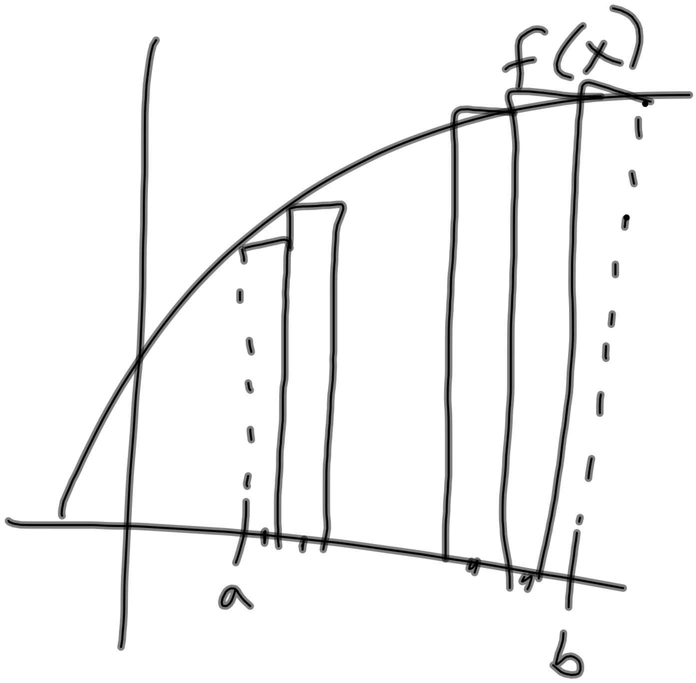
$(b-a)\cdot f(b)$

$\frac{b-a}{2}\cdot f(b)+\frac{b-a}{2}\cdot f\left(a+\frac{b-a}{2}\right)$

upper sum)



lower sum
height of □ is
det. by left endpoint



divide interval into n equal-sized pieces of width $\frac{b-a}{n}$

Lower sum: $s(n) = \sum_{i=1}^{n} f(m_i)\Delta x$

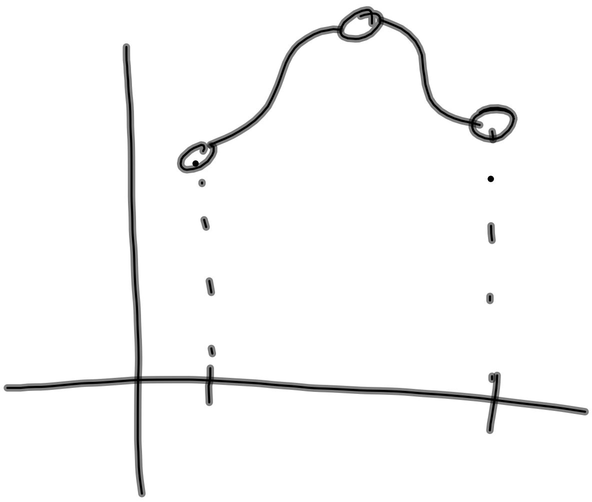
Upper sum: $S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$

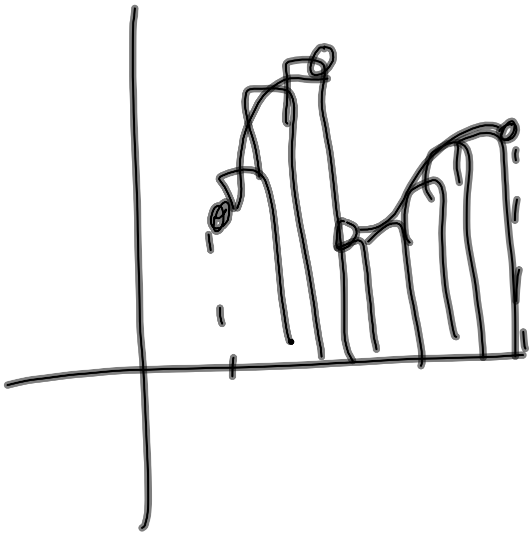
$f(m_i)$ = minimum function value in an interval
$f(M_i)$ = Maximum function value in an interval
$\Delta x = \frac{b-a}{n}$ $\qquad s(n) \leq S(n)$

Area of the region bounded by the graph of f, the x-axis, & the lines $x=a$ & $x=b$ is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x, \quad x_{i-1} \leq c_i \leq x_i$$

where $\Delta x = \frac{b-a}{n}$.

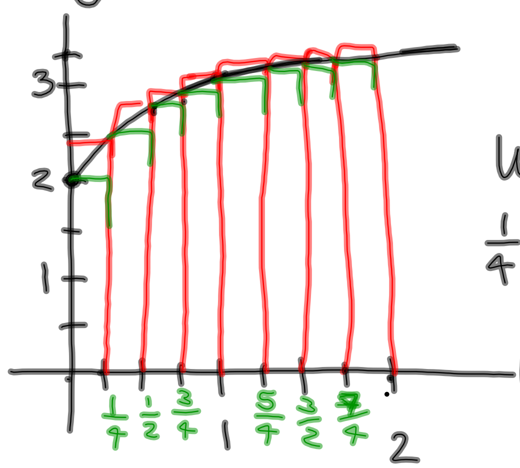




28. $y=\sqrt{x}+2$

$a=0,\ b=2,\ n=8$

$\frac{b-a}{n}=\frac{2-0}{8}=\frac{1}{4}$



Upper sum:

$\frac{1}{4}\left(\sqrt{\frac{1}{4}}+2\right)+\frac{1}{4}\left(\sqrt{\frac{1}{2}}+2\right)+\frac{1}{4}\left(\sqrt{\frac{3}{4}}+2\right)+\dots\frac{1}{4}\left(\sqrt{2}+2\right)$

≈ 6.038

Lower sum:

$\frac{1}{4}\left(\sqrt{0}+2\right)+\dots\frac{1}{4}\left(\sqrt{\frac{7}{4}}+2\right)$

≈ 5.685

actual area $5.685\le A\le 6.038$

$$\lim_{n\to\infty} S(n)$$

32. $S(n) = \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$

$$\lim_{n\to\infty} \frac{\cancel{64}^{32}}{\cancel{6}_{3}}\left(\frac{\cancel{n}(2n^2+3n+1)}{n^{\cancel{3}2}}\right)$$

$$= \lim_{n\to\infty} \frac{32}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) = \boxed{\frac{64}{3}}$$

$$\frac{3}{n} \to 0 \qquad \frac{1}{n^2} \to 0$$

rewrite without summation notation

36. $\sum_{j=1}^{n} \frac{4j+3}{n^2}$

$$= \frac{1}{n^2}\sum_{j=1}^{n} 4j+3$$

$$= \frac{1}{n^2}\sum_{j=1}^{n} 4j + \frac{1}{n^2}\sum_{j=1}^{n} 3$$

$$= \frac{\cancel{4}^{2}}{\cancel{n^2}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{1}{\cancel{n^2}} \cdot 3\cancel{n}$$

$$= \frac{2n+2+3}{n} = \boxed{\frac{2n+5}{n}}$$

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

4.2 Homework

#27 - 37 odd