**Review:** $f(x) = \sqrt[3]{x}$ , $c = 27$

Approximate $\sqrt[3]{28}$ using differentials

$$f(c+\Delta x) = f(c) + f'(c)\,\Delta x$$

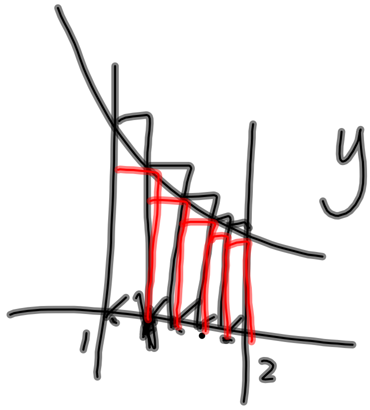
$$\sqrt[3]{27+1} = \sqrt[3]{27} + \frac{1}{3\left(\sqrt[3]{27}\right)^2}(1)$$

$$= 3 + \frac{1}{27}$$

$$= \frac{82}{27} \approx 3.037\ldots$$

## 4.2

$$\Delta x = \frac{b-a}{n}$$

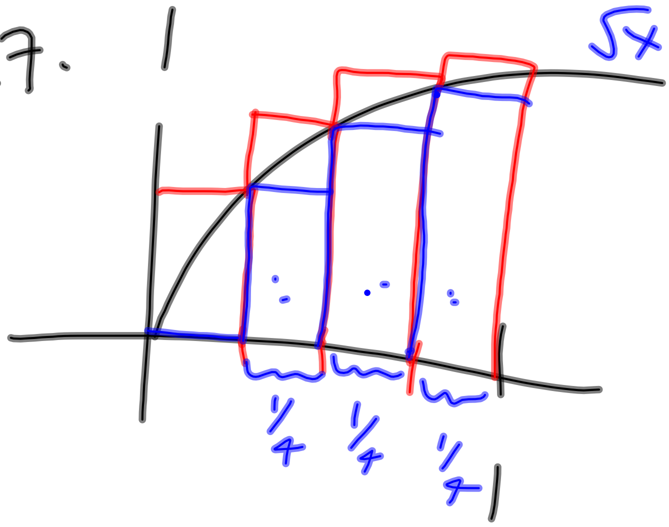



$y = \frac{1}{x}$

29. $\frac{2-1}{5} = \frac{1}{5}$ = width of each rectangle

$$\frac{1}{5}\left(\cancel{\frac{1}{1}} + \frac{1}{1+\frac{1}{5}} + \frac{1}{1+\frac{2}{5}} + \frac{1}{1+3/5} + \frac{1}{1+4/5} + \frac{1}{2}\right)$$

27.




$$\frac{1}{4}\left(\sqrt{1/4}+\sqrt{1/2}+\sqrt{3/4}+\sqrt{1}\right)$$

$$\frac{1}{4}\left(\sqrt{0}+\sqrt{\tfrac{1}{4}}+\sqrt{\tfrac{1}{2}}+\sqrt{\tfrac{3}{4}}\right)$$

33.

$$\frac{\cancel{18}^{\,9}}{n^{\cancel{2}}}\left[\frac{\cancel{n}(n+1)}{\cancel{2}}\right] = \frac{18n^2+\dots}{2n^2+\dots}$$

$$\frac{9n+9}{n} = 9+\frac{9}{n} \to 0 \Rightarrow \boxed{9}$$

37. $\sum_{k=1}^{n} \frac{6k(k-1)}{n^3} = \frac{6}{n^3}\sum_{k=1}^{n} \underbrace{k(k-1)}_{k^2-k}$

$= \frac{6}{n^3}\sum_{k=1}^{n} k^2 - \frac{6}{n^3}\sum_{k=1}^{n} k$

$= \frac{\cancel{6}}{\cancel{n^3}\,2}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}} - \frac{\overset{3}{\cancel{6}}}{\cancel{n^3}\,2}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}}$

$= \frac{2n^2+n+2n+1-3n-3}{n^2} = \boxed{\frac{2n^2-2}{n^2}}$

42. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)$

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right)$

$= \lim_{n\to\infty}\left[\frac{2}{n}\sum_{i=1}^{n}1 + \frac{8}{n^2}\sum_{i=1}^{n}i + \frac{8}{n^3}\sum_{i=1}^{n}i^2\right]$

$= \lim_{n\to\infty}\left[\frac{2}{\cancel{n}}\cdot\cancel{n} + \frac{\overset{4}{\cancel{8}}}{\cancel{n^2}}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{\overset{4}{\cancel{8}}}{\cancel{n^3}\,2}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\underset{3}{\cancel{6}}}\right]$

$2 + 4 + 8/3$

$= \boxed{\frac{26}{3}}$

44. $\lim\limits_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{2i}{n}\right)^3 \left(\frac{2}{n}\right)$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

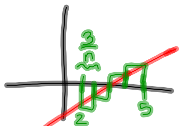
$= \lim\limits_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(1 + \frac{6i}{n} + \frac{12i^2}{n^2} + \frac{8i^3}{n^3}\right)$

$= \lim\limits_{n\to\infty} \left[\frac{2}{n}\sum_{i=1}^{n} 1 + \frac{12}{n^2}\sum_{i=1}^{n} i + \frac{24}{n^3}\sum_{i=1}^{n} i^2 + \frac{16}{n^4}\sum_{i=1}^{n} i^3\right]$

$= \lim\limits_{n\to\infty} \left[\underbrace{\frac{2}{n}\cdot n}_{2} + \underbrace{\frac{12}{n^2}\cdot\frac{n(n+1)}{2}}_{6} + \underbrace{\frac{24}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}}_{8} + \underbrace{\frac{16}{n^4}\cdot\frac{n^2(n+1)^2}{4}}_{4}\right]$

$= \boxed{20}$

48. $y = 3x - 4$ , $[2,5]$



Area $= \lim\limits_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$ , $x_i \le c_i \le x_{i+1}$,

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{5-2}{n} = \frac{3}{n}$

righthand x-coord. of $i^{th}$ rectangle

~~left~~ right endpoint of $i^{th}$ rectangle?

$2 + \frac{3}{n}\cdot 2$
$2 + \frac{3}{n}\cdot 3$
$2 + \frac{3}{n}\cdot i$
$2 + \frac{3i}{n}$

$\lim\limits_{n\to\infty} \sum_{i=1}^{n} \left[3\left(2+\frac{3i}{n}\right) - 4\right]\left(\frac{3}{n}\right)$

$= \lim\limits_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n}\left[6 + \frac{9i}{n} - 4\right]$

$= \lim\limits_{n\to\infty} \left[\frac{6}{n}\sum_{i=1}^{n} 1 + \frac{27}{n^2}\sum_{i=1}^{n} i\right]$

$= \lim\limits_{n\to\infty} \left[\frac{6}{n}\cdot n + \frac{27}{n^2}\cdot\frac{n(n+1)}{2}\right]$

$= 6 + \frac{27}{2} = \boxed{\frac{39}{2}}$

56. $y = x^2 - x^3 \qquad [-1, 0]$

$$\Delta x = \frac{0-(-1)}{n} = \frac{1}{n}$$

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{1}{n}\right)\left[\left(-1+\frac{i}{n}\right)^2 - \left(-1+\frac{i}{n}\right)^3\right]$$

4.2 # 41, 43, 47, 53