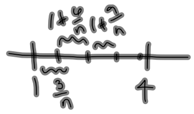
4.2



53. $y = 64 - x^3 \quad [1,4] \quad \Delta x = \frac{b-a}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right)\left[64 - \left(1+\frac{3i}{n}\right)^3\right] =$$

$$\lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n}\left[64 - \left(1 + \frac{9i}{n} + \frac{27i^2}{n^2} + \frac{27i^3}{n^3}\right)\right] =$$

$$\lim_{n\to\infty}\left[\frac{3}{n}\sum_{i=1}^{n} 63 - \frac{27}{n^2}\sum i - \frac{81}{n^3}\sum i^2 - \frac{81}{n^4}\sum i^3\right]$$

$$\lim_{n\to\infty}\left[\frac{3}{n}\cdot 63n - \frac{27}{n^2}\cdot\frac{n(n+1)}{2} - \frac{81}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{81}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right]$$

$$= 189 - \frac{27}{2} - 27 - \frac{81}{4}$$

$$= \frac{513}{4}$$

43. $$\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)\left(\frac{2}{n}\right)$$

$\cancel{y=x}$ $\cancel{[1,3]}$

$$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)$$

$$= \lim_{n\to\infty}\left[\frac{2}{n}\underbrace{\sum_{i=1}^{n} 1}_{n} + \frac{2}{n^2}\underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}}\right]$$

$$= 2 + 1 = \boxed{3}$$

4.3 Riemann Sums & Definite Integrals

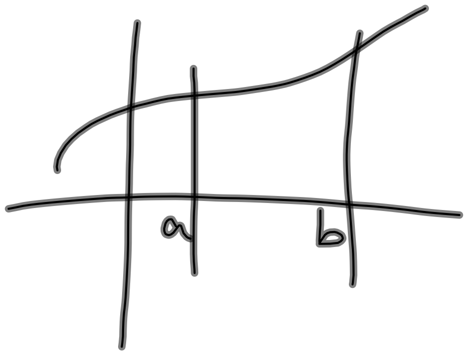
$$\sum_{i=1}^{n} f(c_i)\Delta x_i \quad , \quad x_{i-1} \le c_i \le x_i ,$$

where c_i is any point in the i^{th} subinterval ; $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ is called the Riemann Sum of f.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)\,dx$$

called the definite integral of f from a to b.

Properties



If $f(a)$ is defined,

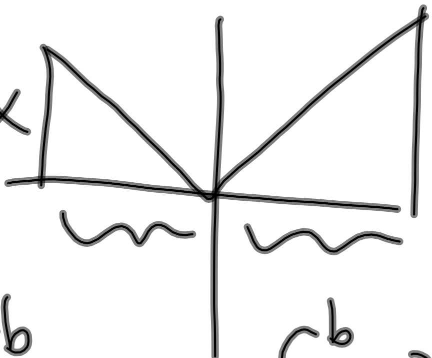
$$\int_a^a f(x)\,dx = 0$$

If f is integrable on $[a,b]$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$$



$$\int_a^b \left[f(x) \pm g(x)\right]dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

If $f(x) \geq 0$,

$$\int_a^b f(x)\,dx \geq 0$$

If $f(x) \leq g(x)$

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

typo on p. 272

8. $\displaystyle\int_{-1}^{2} (3x^2+2)\,dx$

$a=-1,\ b=2$

$\Delta x = \frac{2-(-1)}{n} = \frac{3}{n}$

$f(x) = 3x^2+2$

i^{th} right endpoint: $-1+\frac{3i}{n}$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right)\left(3\left(-1+\frac{3i}{n}\right)^2+2\right)$$

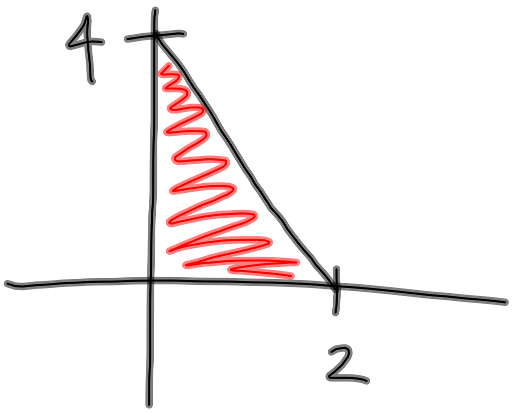
$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[3 - \frac{18i}{n} + \frac{27i^2}{n^2} + 2\right]$$

$$= \lim_{n\to\infty} \left[\frac{3}{n}\sum_{i=1}^{n} 5 - \frac{54}{n^2}\sum_{i=1}^{n} i + \frac{81}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{3}{n}\cdot 5n - \frac{54}{n^2}\cdot\frac{n(n+1)}{2} + \frac{81}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= 15 - 27 + 27 = \boxed{15}$$

14. $f(x)=4-2x$



$$\int_0^2 (4-2x)\,dx$$

40. $\int_2^4 x^3dx = 60$; $\int_2^4 x\,dx = 6$; $\int_2^4 dx = 2$

$$\int_2^4 (6+2x-x^3)\,dx$$

$$= 6\int_2^4 dx + 2\int_2^4 x\,dx - \int_2^4 x^3\,dx$$

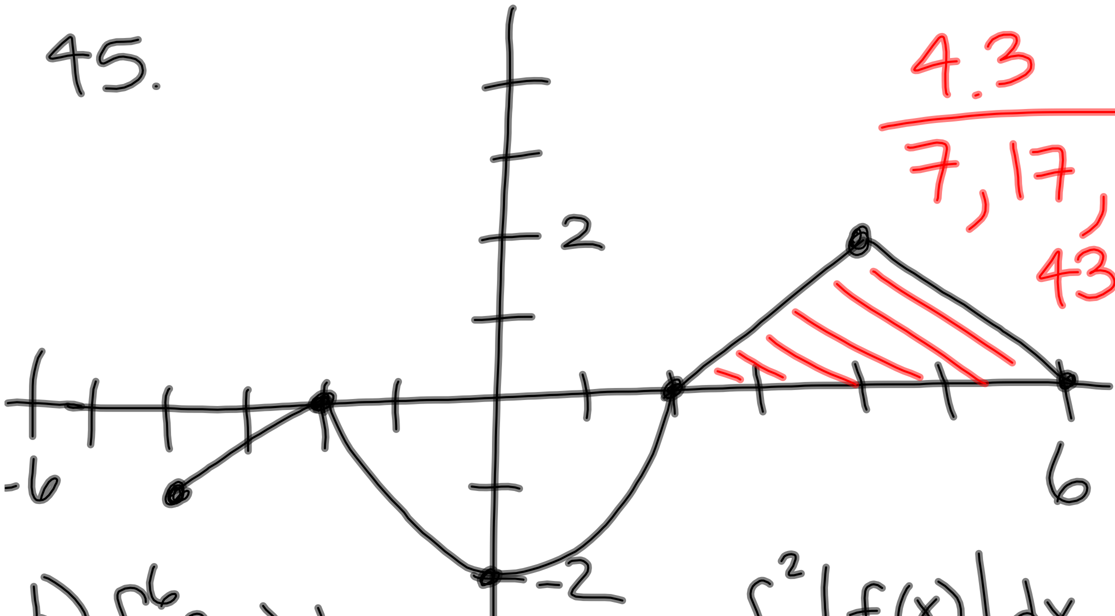
$$= 6(2)+2(6)-60$$

$$= \boxed{-36}$$

45.

4.3

7, 17, 37, 43, 45



b) $\int_{2}^{6} f(x)\,dx$

$= \frac{1}{2}(4)(2) = \boxed{4}$

$\int_{-2}^{2} |f(x)|\,dx$

$= \frac{1}{2}\pi(2)^2 = \boxed{2\pi}$