9. $f''(x) = 6(x-1)$; $(2,1)$
$= 6x - 6$

tangent to $3x - y - 5 = 0$ @

$y = \boxed{3}x - 5$

$f'(x) = \underbrace{3x^2 - 6x + c_1}$ $= 3$ when $x = 2$

$\Rightarrow c_1 = 3$

$f(x) = x^3 - 3x^2 + c_1 x + C_2$

$f(x) = x^3 - 3x^2 + 3x + C_2$

$1 = 2^3 - 3(2)^2 + 3(2) + C_2$

$1 = 8 - 12 + 6 + C_2$

$-1 = C_2$

particular solution:

$$\boxed{f(x) = x^3 - 3x^2 + 3x - 1}$$

12. acceleration $a(t) = \sin t$, $t > 0$
position $v(0) = 0$
$x(0) = 5$

(a) $v(t) =$ & $x(t)$
(b) find t s.t. $v(t) = 0$

(a) $v(t) = -\cos t + C$
$0 = -\cos(0) + C$
$C = 1$

$$\boxed{v(t) = -\cos t + 1}$$

$x(t) = -\sin t + t + C_2$
$5 = -\sin(0) + 0 + C_2$
$C_2 = 5$

$$\boxed{x(t) = -\sin t + t + 5}$$

(b) $v(t) = -\cos t + 1$
$0 = -\cos t + 1$
$\cos t = 1$

$$\boxed{t = 2\pi k \ , k \geq 0}$$

11. $\frac{dP}{dt} = k\sqrt[3]{t} = kt^{1/3}$ $\quad 0 \le t \le 10$

$P(0) = 1000$; $P(1) = 1100$

$P(10) = ?$

$P = k \cdot \frac{3}{4} t^{4/3} + C$

$1000 = k \cdot \frac{3}{4}(0)^{4/3} + C \Rightarrow C = 1000$

$1100 = k \cdot \frac{3}{4}(1)^{4/3} + C$

$100 = \frac{3k}{4}$

$k = \frac{400}{3}$

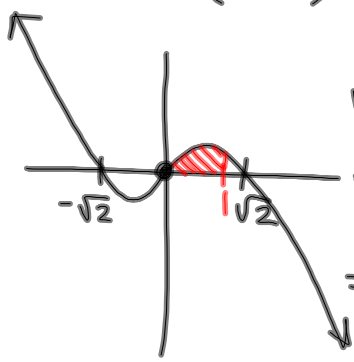
$P(t) = 100t^{4/3} + 1000$

$P(10) = 100\sqrt[3]{10^4} + 1000$

≈ 3154 bacteria

17. $y = 2x - x^3$, $[0, 1]$

$= x(2 - x^2)$



$\lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(\frac{i}{n}\right) - \left(\frac{i}{n}\right)^3 \right] \cdot \frac{1}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{2}{n^2} i - \frac{i^3}{n^4} \right]$

$= \lim_{n\to\infty} \left[\frac{2}{n^2} \sum_{i=1}^{n} i - \frac{1}{n^4} \sum_{i=1}^{n} i^3 \right]$

$= \lim_{n\to\infty} \left[\frac{2}{n^2} \cdot \frac{n(n+1)}{2} - \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4} \right]$

$= 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$

$\int_0^1 (2x - x^3)\,dx = x^2 - \frac{1}{4}x^4 \Big|_0^1 = 1 - \frac{1}{4} = \frac{3}{4}$ ✓

3. use differentials to approximate $\sqrt[3]{63}$

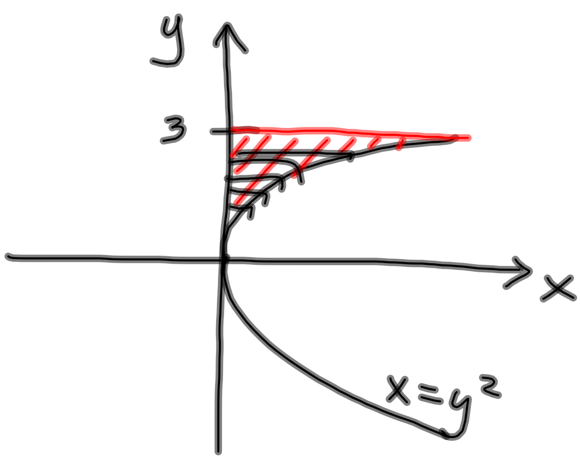
$$f(c+\Delta x) \approx f(c) + f'(c)\Delta x$$

$$\begin{cases} c = 64 \\ f(x) = \sqrt[3]{x} = x^{1/3} \\ f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}} \\ \Delta x = -1 \end{cases}$$

$$\sqrt[3]{63} \approx \sqrt[3]{64} + \frac{1}{3(\sqrt[3]{64})^2}\cdot(-1)$$

$$= \boxed{4 + \frac{-1}{48}}$$

19. $f(y) = y^2$, $0 \le y \le 3$



$$\int_0^3 y^2\,dy$$

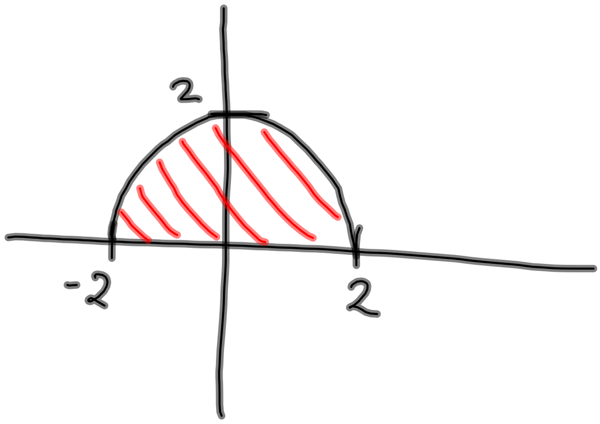
$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^2 \cdot \frac{3}{n}$$

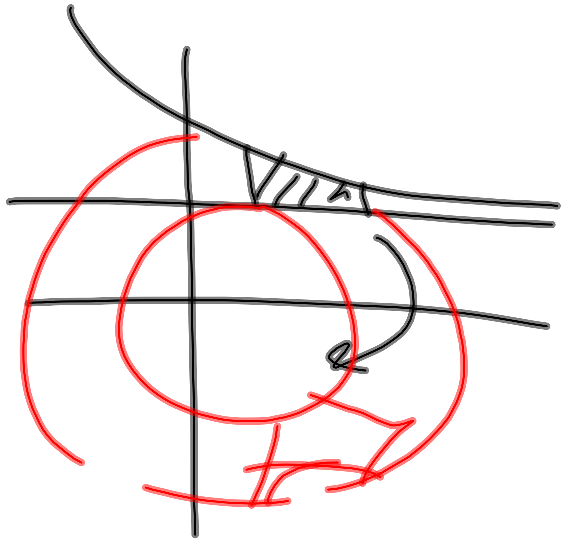
$$= \lim_{n\to\infty} \frac{27}{n^3} \sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \boxed{9}$$

23. $\int_{-2}^{2} \sqrt{4-x^2}\,dx = \dfrac{\pi(2)^2}{2} = \boxed{2\pi}$





18. $y = x^2 - x^3 \quad , \quad [-1, 0]$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-1+\frac{i}{n}\right)^2 - \left(-1+\frac{i}{n}\right)^3\right] \cdot \frac{0-(-1)}{n}$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left[\left(1 - \frac{2i}{n} + \frac{i^2}{n^2}\right) - \left(-1 + 3\frac{i}{n} - 3\frac{i^2}{n^2} + \frac{i^3}{n^3}\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\sum_{i=1}^{n} 1 - \frac{5}{n^2}\sum_{i=1}^{n} i + \frac{4}{n^3}\sum_{i=1}^{n} i^2 - \frac{1}{n^4}\sum_{i=1}^{n} i^3\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n} \cdot \frac{n}{1} - \frac{5}{n^2} \cdot \frac{n(n+1)}{2} + \frac{4}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4}\right]$$

$$= 2 - \frac{5}{2} + \frac{4}{3} - \frac{1}{4}$$

$$= \frac{24}{12} - \frac{30}{12} + \frac{16}{12} - \frac{3}{12}$$

$$= \boxed{\frac{7}{12}}$$

$$f(c+\Delta x) = f(c) + f'(c)\Delta x$$

$$\sum c \quad \sum i \quad \sum i^2 \quad \sum i^3$$

$$\lim \sum f\left(a + \frac{b-a}{n} i\right) \cdot \frac{b-a}{n} =$$

$$\int_a^b f(x)\,dx$$

$$\int_a^b = \int_a^c + \int_c^b \quad ; \quad \int_a^b = -\int_b^a$$