4.4

45. $f(x)=x-2\sqrt{x}$, $[0,2]$

$$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$$

$$\int_0^2 (x-2x^{1/2})\,dx = [c-2\sqrt{c}](2-0)$$

$$\frac{1}{2}x^2-\frac{4}{3}x^{3/2}\Big|_0^2 = 2c-4\sqrt{c}$$

$$\frac{1}{2}(2)^2-\frac{4}{3}(\sqrt{2^3}) = 2c-4\sqrt{c}$$

$$\boxed{2-\frac{8\sqrt{2}}{3} = 2c-4\sqrt{c}}$$

F2 Algebra
Solve $(2-(8/3)\cdot\sqrt{(2)}=2x-4\sqrt{(x)}, x)$

$\boxed{x=0.4 \text{ or } 1.79}$

$$1-\frac{4\sqrt{2}}{3} = c-2\sqrt{c}$$

$$\left(1-\frac{4\sqrt{2}}{3}-c\right)^2=\left(-2\sqrt{c}\right)^2$$

$$\left(1-\frac{4\sqrt{2}}{3}\right)^2-2\left(1-\frac{4\sqrt{2}}{3}\right)c+c^2=4c$$

$$c^2+\left[-2\left(1-\frac{4\sqrt{2}}{3}\right)-4\right]c+\left(1-\frac{4\sqrt{2}}{3}\right)^2=0$$

$$c=\frac{-\left[-2\left(1-\frac{4\sqrt{2}}{3}\right)-4\right]\pm\sqrt{\left[-2\left(1-\frac{4\sqrt{2}}{3}\right)-4\right]^2-4(1)\left(1-\frac{4\sqrt{2}}{3}\right)^2}}{2}$$

$=$

Who cares?!

47. $f(x)=2\sec^2 x$, $[-\pi/4, \pi/4]$

$$\int_{-\pi/4}^{\pi/4} 2\sec^2 x\,dx = [2\sec^2(c)]\cdot\left(\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right)$$

$$2\tan\frac{\pi}{4}-2\tan\left(\frac{-\pi}{4}\right) = \pi\sec^2 c$$

$$2-2(-1) = \pi\sec^2 c$$

$$\boxed{\frac{4}{\pi} = \sec^2 c}$$

solve$(4/\pi=(\sec(x))^2, x)$

$$\sec c = \pm\sqrt{\frac{4}{\pi}}$$

$$c = \sec^{-1}\left(\pm\sqrt{\frac{4}{\pi}}\right)$$

$$= \cos^{-1}\left(\pm\frac{1}{\sqrt{4/\pi}}\right)$$

$\boxed{x \pm 0.48}$

81. $F(x)=\int_{-2}^{x}(t^2-2t)\,dt$

$F'(x)=\boxed{x^2-2x}$

$F(x)=\int_{a}^{x}f(t)\,dt$

$\Rightarrow F'(x)=f(x)$

$F(x)=\int_{-2}^{x^2}(t^2-2t)\,dt$

$F(x)=\int_{a}^{g(x)}f(t)\,dt$

$F'(x)=\left[(x^2)^2-2(x^2)\right]\cdot 2x$

$F'(x)=f(g(x))\cdot g'(x)$

89. $F(x)=\int_{0}^{\sin x}\sqrt{t}\,dt$

$F'(x)=\sqrt{\sin x}\cdot\cos x$

$=\boxed{\cos x\sqrt{\sin x}}$

4.5

50. $\int \sqrt{\tan x}\ \sec^2 x\, dx = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C$

$u = \tan x$

$du = \sec^2 x\, dx$

$= \boxed{\frac{2}{3}(\tan x)^{3/2} + C}$

54. $\int \csc^2\left(\frac{x}{2}\right) dx = 2\int \csc^2 u\, du$

$u = \frac{x}{2}$

$\frac{du}{1/2} = \frac{\frac{1}{2}dx}{1/2}$

$2du = dx$

$= -2\cot u + C$

$= \boxed{-2\cot\left(\frac{x}{2}\right) + C}$

52. $\int \frac{\sin x}{\cos^3 x} dx = \int \frac{-du}{u^3} = \int -u^{-3} du$

$u = \cos x$

$du = -\sin x\, dx$

$= \frac{1}{2u^2} + C$

$\int \tan x \sec^2 x\, dx$

$= \boxed{\frac{1}{2\cos^2 x} + C}$

58. $\int x\sqrt{2x+1}\,dx = \int \left(\frac{u-1}{2}\right)\sqrt{u}\cdot\frac{1}{2}du =$

$u = 2x+1 \longleftrightarrow \frac{u-1}{2} = x$

$u\sqrt{u} = u^1 \cdot u^{1/2} = u^{3/2}$

$\frac{du}{2} = \frac{\cancel{2}dx}{\cancel{2}}$

$\frac{1}{2}du = dx$

$= \frac{1}{4}\int\left(u^{3/2} - u^{1/2}\right)du$

$= \frac{1}{4}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$

$= \boxed{\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + C}$

62. $\int \frac{2x+1}{\sqrt{x+4}}dx = \int \frac{2(u-4)+1}{\sqrt{u}}du$

$u = x+4 \leftrightarrow x = u-4$

$du = dx$

$= \int (2u^1 - 7)u^{-1/2}\,du$

$= \int \left(2u^{1/2} - 7u^{-1/2}\right)du$

$= \frac{4}{3}u^{3/2} - 14u^{1/2} + C$

$= \boxed{\frac{4}{3}(x+4)^{3/2} - 14(x+4)^{1/2} + C}$

Definite Integrals

$\int u^2 = \frac{1}{3}u^3$

66. $\int_{-2}^{4} x^2(x^3+8)^2 dx = \int \frac{1}{3}u^2 du$

$u = x^3+8$

$\frac{du}{3} = \frac{\cancel{3}x^2 dx}{\cancel{3}}$

$= \frac{1}{9}u^3 = \frac{1}{9}(x^3+8)^3 \Big|_{-2}^{4}$

$= \frac{1}{9}[4^3+8]^3 - \cancel{\frac{1}{9}[(-2)^3+8]^3} \rightarrow 0$

$= \frac{72^3}{9} = \boxed{41472}$

4.5 # 7-33, 41-53,

57-63 odd