Quiz #2

2^{nd} FTC :

$$F(x) = \int_a^x f(t)\,dt$$

$$F'(x) = f(x)$$

$$F(x) = \int_a^{g(x)} f(t)\,dt$$

$$F'(x) = f(g(x)) \cdot g'(x)$$

1. $F(x) = \int_0^x \sec^3 t\,dt$

$$F'(x) = \boxed{\sec^3 x}$$

2. $F(x) = \int_0^{\sin x} \sqrt{t}\,dt$

$$F'(x) = \boxed{\sqrt{\sin x} \cdot \cos x}$$

$$F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$$

$$= \int_{g(x)}^{a} f(t)\,dt + \int_a^{h(x)} f(t)\,dt$$

$$= -\int_a^{g(x)} f(t)\,dt + \int_a^{h(x)} f(t)\,dt$$

$$F'(x) = f(h(x)) \cdot h'(x) - f(g(x)) \cdot g'(x)$$

3. $\int \frac{x^2}{(1+x^3)^2}dx = \int \frac{\frac{1}{3}du}{u^2} = \int \frac{1}{3}u^{-2}du$

$u = 1+x^3$

$\frac{du}{3} = \frac{\cancel{3}x^2dx}{\cancel{3}}$

$= -\frac{1}{3}u^{-1} = \boxed{-\frac{1}{3}(1+x^3)^{-1}+C}$

$= \boxed{\frac{-1}{3(1+x^3)}+C}$

$= \boxed{\frac{-1}{3+3x^3}+C}$

4. $\int \tan^4 x \sec^2 x\, dx = \int u^4 du$

$u = \tan x$

$du = \sec^2 x\, dx$

$= \frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{5}\tan^5 x + C}$

5. $\int_1^2 2x^2\sqrt{x^3+1}\,dx = \int_{x=1}^{x=2} \frac{2}{3}u^{1/2}\,du$

$u = x^3+1$

$2\cdot\frac{du}{3} = \frac{\cancel{3}x^2\,dx}{\cancel{3}}\cdot 2$

$= \frac{2}{3}\cdot\frac{2}{3}\cdot u^{3/2}\Big|_{x=1}^{x=2}$

$\sqrt{u^3}$

$= \frac{4}{9}\sqrt{(2^3+1)^3} - \frac{4}{9}\sqrt{(1^3+1)^3}$

$= \boxed{12 - \frac{8\sqrt{2}}{9}}$

≈ 10.7

4.5 #63

$\int \frac{-x}{(x+1)-\sqrt{x+1}}\,dx = \int \frac{-(u^2-1)\cdot 2u\,du}{u^2-u}$

$u = \sqrt{x+1} = (x+1)^{1/2}$

$du = \frac{1}{2}(x+1)^{-1/2}\,dx$

$du = \frac{dx}{2\sqrt{x+1}}$

$dx = 2\sqrt{x+1}\,du$

$dx = 2u\,du$

$u^2 = x+1$

$u^2-1 = x$

$= \int \frac{-2\cancel{u}(u+1)\cancel{(u-1)}\,du}{\cancel{u}\cancel{(u-1)}}$

$= \int(-2u-2)\,du$

$= -u^2 - 2u + C$

$= \boxed{-(x+1) - 2\sqrt{x+1} + C}$

5.2

34. $\int \frac{\csc^2 t}{\cot t}\,dt = \int \frac{-1}{u}\,du$

$u = \cot t$

$\frac{du}{-1} = \frac{\cancel{-}\csc^2 t\,dt}{\cancel{-1}}$

$= -\ln|u| + c$

$= -\ln|\cot t| + c$

64. $F(x) = \int_1^{x^2} \frac{1}{t}\,dt$

Find $F'(x)$.

$F'(x) = \frac{1}{x^2} \cdot 2x$

5.4

$$\int e^x dx = e^x + C$$

$[e^x]' = e^x$

5.5

$[a^x]' = a^x \cdot \ln a$

$$\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$$

5.4

94. $\displaystyle\int \frac{e^{1/x^2}}{x^3} dx = \int -\frac{1}{2} e^u du$

$u = \frac{1}{x^2} = x^{-2}$

$\frac{du}{-2} = -2x^{-3} dx = \frac{\frac{\cancel{-2}dx}{x^3}}{\cancel{-2}}$

$= -\frac{1}{2} e^u + C$

$= \boxed{-\frac{1}{2} e^{1/x^2} + C}$

104. $\int \frac{e^{2x}+2e^{x}+1}{e^{x}}\,dx$

$= \int \left(e^{x} + 2 + e^{-x}\right)dx$

$u=-x$

$\frac{du}{-1} = \frac{\cancel{-}dx}{\cancel{-1}}$

$\int -e^{u}\,du$

$= \boxed{e^{x} + 2x - e^{-x} + C}$

108. $\int \ln(e^{2x-1})\,dx = \int (2x-1)\,dx$

$= \boxed{x^{2} - x + C}$

5.5

68. $\int 2^{\sin x} \cos x\,dx = \int 2^u\,du$

$u = \sin x$
$du = \cos x\,dx$

$= \frac{1}{\ln 2} \cdot 2^u + C$

$= \boxed{\frac{2^{\sin x}}{\ln 2} + C}$

64. $\int_{-2}^{0} (3^3 - 5^2)\,dx = \int_{-2}^{0} 2\,dx$

$= 2x \Big|_{2}^{0} = 2(0) - 2(-2) = \boxed{4}$

5.4

102. $\int \frac{2e^{x}-2e^{-x}}{(e^{x}+e^{-x})^{2}}\,dx = \int \frac{2du}{u^{2}}$

$u = e^{x}+e^{-x}$

$2du = 2(e^{x}-e^{-x})\,dx$

$= \int 2u^{-2}\,du$

$= -2u^{-1}+c$

$= \boxed{\frac{-2}{e^{x}+e^{-x}}+c}$

5.2

19-35, 43-53, 61, 63

5.4

87-107

5.5

61-67