5.2

31. $\int \csc 2x\,dx = \int \frac{1}{2}\csc u\,du$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = \frac{\cancel{2}dx}{\cancel{2}}$

$= -\frac{1}{2}\ln|\csc u + \cot u| + C$

$= -\frac{1}{2}\ln|\csc 2x + \cot 2x| + C$

47. $\int_0^2 \frac{x^2-2}{x+1}\,dx = \int_{x=0}^{x=2} \frac{(u-1)^2-2}{u}\,du$

$u = x+1 \quad x = u-1$

$du = dx$

$\int_{x=0}^{x=2} \frac{u^2-2u-1}{u}\,d\cancel{x} = \int_{x=0}^{2} \left(u-2-\frac{1}{u}\right)du$

$\frac{u^2}{2} - 2u - \ln|u| \Big|_{x=0}^{2} = \frac{(x+1)^2}{2} - 2(x+1) - \ln|x+1| \Big|_{x=0,2}$

$\frac{(2+1)^2}{2} - 2(2+1) - \ln|2+1| - \left[\frac{(0+1)^2}{2} - 2(0+1) - \ln|0+1|\right]$

$= \frac{9}{2} - 6 - \ln 3 - \frac{1}{2} + 2$

$\boxed{= -\ln 3}$

53. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\log(a^p) = p \cdot \log a$

$\int \sec x\,dx = -\ln|\sec x - \tan x| + C$

$= \ln|\sec x - \tan x|^{-1} + C$

$$\frac{1}{\sec x - \tan x} = \frac{1}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}} = \frac{1}{\frac{1-\sin x}{\cos x}} =$$

$$\frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x(1+\sin x)}{1-\sin^2 x}$$

$$= \frac{\cancel{\cos x}(1+\sin x)}{\cos^{\cancel{2}} x} = \frac{1+\sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \sec x + \tan x$$

63. $F(x) = \int_x^{3x} \frac{1}{t}\,dt$

$$= \int_x^a \frac{1}{t}\,dt + \int_a^{3x} \frac{1}{t}\,dt$$

$$= \int_a^{3x} \frac{1}{t}\,dt - \int_a^{x} \frac{1}{t}\,dt$$

$$F'(x) = \frac{1}{3x} \cdot 3 - \frac{1}{x} = \boxed{0}$$

45. $\int_1^e \frac{(1+\ln x)^2}{x}\,dx = \int_{x=1}^{e} u^2\,du$

$u = 1+\ln x$

$du = \frac{1}{x}\,dx$

$= \frac{u^3}{3} = \frac{(1+\ln x)^3}{3}$

$\frac{(1+\ln e)^3}{3} - \frac{(1+\ln 1)^3}{3}$

$\frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}$

$(\arctan x)' = \frac{1}{1+x^2}$; $\int \frac{dx}{1+x^2} = \arctan x + C$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$; $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$

$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$; $\int \frac{dx}{|x|\sqrt{x^2-1}} = \operatorname{arcsec} x + C$