5.4

$$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$$

103. $\int \frac{5-e^x}{e^{2x}}\,dx$

$$=\int\left(5e^{-2x}-e^{-x}\right)dx=\int 5e^{-2x}dx+\int -e^{-x}dx$$

$u=-2x$

$-\frac{5}{2}\cdot du=\cancel{-2}dx\cdot\frac{-5}{\cancel{-2}}$

$v=-x$

$dv=-dx$

$$=\int -\frac{5}{2}e^{u}du+\int e^{v}du$$

$$=-\frac{5}{2}e^{u}+e^{v}+C=\boxed{-\frac{5}{2}e^{-2x}+e^{-x}+C}$$

5.5

65. $\int x\left(5^{-x^2}\right)dx=\int -\frac{1}{2}\cdot 5^{u}du$

$u=-x^2$

$\frac{du}{-2}=\frac{\cancel{-2}x\,dx}{\cancel{-2}}$

$$=\boxed{-\frac{1}{2\ln 5}5^{-x^2}+C}$$

5.4

97. $\displaystyle\int_1^3 \frac{e^{3/x}}{x^2}\,dx = \int_{x=1}^{3} -\frac{1}{3}e^u\,du$

$u = \frac{3}{x}$

$\frac{du}{-3} = \frac{\cancel{-3}}{x^2}dx$

$\displaystyle -\frac{1}{3}e^{3/x}\Big|_1^3$

$= \boxed{-\frac{1}{3}e + \frac{1}{3}e^3}$

5.9 Inverse Trig Functions

$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$ | $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$ | $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

$\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$ | $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

2. $\int \frac{3dx}{\sqrt{1-4x^2}} = \frac{3}{2}\int \frac{du}{\sqrt{a^2-u^2}}$

$a=1$ $u=2x$

$a^2=1^2$ $u^2=(2x)^2$

$3\cdot\frac{du}{2}=\frac{\cancel{2}dx}{\cancel{2}}\cdot 3$

$= \frac{3}{2}\arcsin\frac{u}{a}+c$

$= \boxed{\frac{3}{2}\arcsin 2x+c}$

8. $\int_{\sqrt{3}}^{3}\frac{1}{9+x^2}dx = \int_{x=\sqrt{3}}^{3}\frac{du}{a^2+u^2}$

$a=3$ $u=x$

$= \frac{1}{a}\arctan\frac{u}{a}+c$

$= \frac{1}{3}\arctan\frac{x}{3}\Big|_{x=\sqrt{3}}^{3}$

$= \frac{1}{3}\arctan 1 - \frac{1}{3}\arctan\frac{\sqrt{3}}{3}$

$= \frac{1}{3}\cdot\frac{\pi}{4} - \frac{1}{3}\cdot\frac{\pi}{6}$

$= \frac{\pi}{12} - \frac{\pi}{18}$

$= \frac{2\pi}{72} = \boxed{\frac{\pi}{36}}$

12. $\int \frac{x^4-1}{x^2+1}dx = \int \frac{\cancel{(x^2+1)}(x^2-1)}{\cancel{x^2+1}}dx$

$= \int (x^2-1)dx = \boxed{\frac{1}{3}x^3 - x + C}$

16. $\int \frac{1}{x\sqrt{x^4-4}}dx = \int \frac{x\,dx}{x^2\sqrt{x^4-4}}$

$(x^2)^2 - 2^2$

$u = x^2$

$\frac{du}{2} = \frac{\cancel{2}x\,dx}{\cancel{2}}$

$= \frac{1}{2}\int \frac{du}{u\sqrt{u^2-2^2}}$

$= \boxed{\frac{1}{4}\operatorname{arcsec}\frac{x^2}{2} + C}$

30. $\int \frac{x-2}{(x+1)^2+4}dx = \int \frac{x\,dx}{\cancel{(x+1)^2+4}} - \int \frac{2dx}{(x+1)^2+4}$

x^2+2x+5

$= -2\left[\frac{1}{2}\arctan\frac{x+1}{2}\right] + \int \frac{x\,dx}{x^2+2x+5}$

$u = x+1 \quad x = u-1$

$du = dx$

$\cancel{\int (x^2+2x+5)^{-1} x\,dx}$

$\int \frac{(u-1)du}{u^2+4} = \underbrace{\int \frac{u\,du}{u^2+4}} + \int \frac{-du}{u^2+4}$

$-\frac{1}{2}\arctan\frac{x+1}{2}$

$v = u^2+4$

$\frac{dv}{2} = \frac{\cancel{2}u\,du}{\cancel{2}}$

$\int \frac{1}{2}\frac{dv}{v} = \frac{1}{2}\ln|v|$

$= \frac{1}{2}\ln(u^2+4)$

$= \frac{1}{2}\ln[(x+1)^2+4]$

$= -\arctan\frac{x+1}{2} + \frac{1}{2}\ln[(x+1)^2+4] - \frac{1}{2}\arctan\frac{x+1}{2}$

$= \boxed{-\frac{3}{2}\arctan\frac{x+1}{2} + \frac{1}{2}\ln[(x+1)^2+4] + C}$

<u>5.9</u> #1-29 odd