5.9

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+c$$

13. $$\int \frac{1}{\sqrt{1-(x+1)^2}}\,dx = \int \frac{du}{\sqrt{a^2-u^2}} =$$

$a=1$ $u=x+1$

$du=dx$

$$=\arcsin\frac{u}{a}+c$$

$$=\boxed{\arcsin(x+1)+c}$$

$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = -\arccos x + c$$

5.9

$$\frac{x^2}{2} \qquad = \int -\frac{1}{2}\frac{du}{u} = -\frac{1}{2}\ln|u$$

11. $$\int \frac{x^3\,dx}{x^2+1} = \int x\,dx + \int \frac{-x}{x^2+1}\,dx$$

$$\begin{array}{r|l} & x + \frac{-x}{x^2+1} \\ \hline x^2+1 & x^3 \\ & -(x^3+x) \\ \hline & -x \end{array}$$

$u=x^2+1$

$$\frac{du}{-2} = \frac{2x\,dx}{-2}$$

$$-\frac{1}{2}du = -x\,dx$$

$$=\boxed{\frac{x^2}{2} - \frac{1}{2}\ln(x^2+1)+C}$$

$$17.\ \int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = \int_0^{1/\sqrt{2}} \left(\frac{dx}{\sqrt{1-x^2}}\right)(\arcsin x)$$

$u = \arcsin x$

$du = \dfrac{dx}{\sqrt{1-x^2}}$

$$= \int_{x=0}^{1/\sqrt{2}} u\,du = \frac{u^2}{2} = \left.\frac{(\arcsin x)^2}{2}\right|_0^{1/\sqrt{2}}$$

$$\frac{(\arcsin 1/\sqrt{2})^2}{2} - \frac{(\arcsin 0)^2}{2} = \frac{\left(\frac{\pi}{4}\right)^2}{2} = \boxed{\frac{\pi^2}{32}}$$

$$19.\ \int_{-1/2}^{0} \frac{x}{\sqrt{1-x^2}}\,dx = \int_{x=-1/2}^{0} -\frac{1}{2}\cdot u^{-1/2}\,du$$

$u = 1-x^2$

$\dfrac{du}{-2} = \dfrac{\cancel{-2}x\,dx}{\cancel{-2}}$

$$= -u^{1/2}\Big|_{x=-1/2}^{0} =$$

$$= -\sqrt{1-x^2}\Big|_{-1/2}^{0}$$

$$= -\sqrt{1-0^2} - \left(-\sqrt{1-(-1/2)^2}\right)$$

$$= -1 + \sqrt{3/4} = \boxed{-1+\sqrt{3}/2}$$

21. $$\int \frac{e^{2x}}{4+e^{4x}}\,dx = \int \frac{1}{2}\,\frac{du}{4+u^2}$$

$a=2$

$u = e^{2x}$

$$\frac{du}{2} = \frac{\cancel{2}e^{2x}\,dx}{\cancel{2}}$$

$$= \frac{1}{2}\cdot\frac{1}{2}\arctan\frac{e^{2x}}{2} + C$$

$$= \boxed{\frac{1}{4}\arctan\frac{e^{2x}}{2} + C}$$

32. $$\int_{-2}^{2} \frac{dx}{x^2+4x+13}$$ $$\int \frac{du}{a^2+u^2}$$

completing the square:

$$ax^2+bx+c \sim A(x-h)^2+K$$

$$a\left(x^2+\frac{b}{a}x\right)+c$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$a\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2\right)+c-\frac{b^2}{4a}$$

$$(a-b)^2 = a^2-2ab+b^2$$

$$a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a}$$

32. $\int_{-2}^{2} \frac{dx}{x^2+4x+13} = \int_{-2}^{2} \frac{dx}{(x+2)^2+9} = \int_{x=-2}^{2} \frac{du}{u^2+a^2}$

$u=x+2$ $a=3$

$du=dx$

$(x^2+4x)+13$

$(x^2+4x+4)-4+13$

$(x+2)^2+9$

$= \frac{1}{3}\arctan\frac{x+2}{3}\Big|_{x=-2}^{2}$

$= \frac{1}{3}\arctan\frac{4}{3} - \cancel{\frac{1}{3}\arctan 0}$

$= \boxed{\frac{1}{3}\arctan\frac{4}{3}} \approx \boxed{0.309}$

42. $\int \frac{x}{\sqrt{9+8x^2-x^4}}\,dx = \int \frac{x\,dx}{\sqrt{25-(x^2-4)^2}}$

$a=5$ $u=x^2-4$

$\frac{du}{2} = \frac{\cancel{2}x\,dx}{\cancel{2}}$

$-(x^4-8x^2)+9$

$a^2-2ab+b^2=(a-b)^2$

$-(x^4-8x^2+16)+9+16$

$-(x^2-4)^2+25$

$= \int \frac{1}{2}\frac{du}{\sqrt{a^2-u^2}}$

$= \boxed{\frac{1}{2}\arcsin\frac{x^2-4}{5}+C}$

40. $$\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$$

$$x^2-2x = x^2-2x+1-1$$
$$= (x-1)^2-1$$

$u=x-1, du=dx, a=1$

$$= \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{1}\operatorname{arcsec}\frac{|x-1|}{1} + C$$

$$= \boxed{\operatorname{arcsec}|x-1| + C}$$

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100. $$\int \frac{1}{3+25x^2}\,dx = \int \frac{1}{5}\frac{du}{a^2+u^2}$$

$$(\sqrt{3})^2+(5x)^2$$

$u=5x$

$$\frac{du}{5}=\frac{\cancel{5}dx}{\cancel{5}}$$

$$= \boxed{\frac{1}{5}\cdot\frac{1}{\sqrt{3}}\arctan\frac{5x}{\sqrt{3}} + C}$$

104. $$\int \frac{4-x\,dx}{\sqrt{4-x^2}}$$

$$= \underline{\int \frac{4dx}{\sqrt{4-x^2}}} + \underline{\int \frac{-xdx}{\sqrt{4-x^2}}}$$

$u=4-x^2$

$\frac{du}{2}=\frac{-2xdx}{2}$

$\frac{1}{2}du=-xdx$

$$4\arcsin\frac{x}{2} + \int \frac{1}{2}u^{-1/2}du + C$$

$$= \frac{1}{2}u^{1/2}\cdot\frac{2}{1}$$

$$\boxed{=4\arcsin\frac{x}{2}+\sqrt{4-x^2}+C}$$

105. $$\int \frac{\arctan(x/2)}{4+x^2}dx$$

$u=\arctan\frac{x}{2}$

$du=\frac{1}{\frac{4}{4}\cdot 1+\frac{x^2}{4}}\cdot\frac{1}{2}dx$

$=\frac{1}{\frac{4+x^2}{4}}\cdot\frac{1}{2}dx$

$\frac{du}{2}=\frac{\cancel{2}\,1}{4+x^2}dx$

$$=\int\frac{1}{2}udu$$

$$\boxed{=\frac{1}{4}\left(\arctan\frac{x}{2}\right)^2+C}$$

HW:

Take-Home Quiz #2

5.9 #31-41 odd