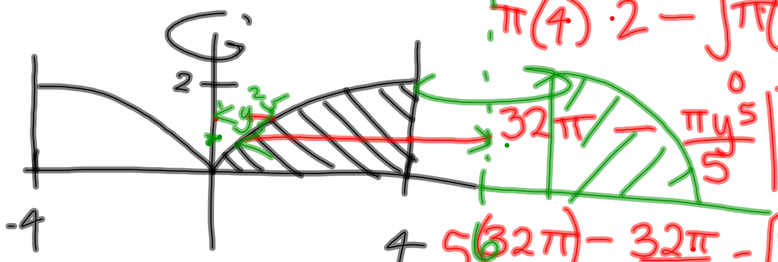
6.2

11. $y=\sqrt{x}$ , $y=0$ , $x=4$

(b)




$\pi(4)^2\cdot 2-\int_0^2 \pi(y^2)^2\,dy$

$=32\pi-\frac{\pi y^5}{5}\Big|_0^2$

$\frac{5(32\pi)}{5}-\frac{32\pi}{5}=\boxed{\frac{128\pi}{5}}$

(d) about $x=6$

$\int_0^2 \pi(6-y^2)^2\,dy-\pi(2)^2\cdot 2$

$=\pi\int_0^2(36-12y^2+y^4)\,dy-8\pi$

$=36\pi y-4y^3\pi+\frac{\pi y^5}{5}\Big|_0^2-8\pi$

$=72\pi-32\pi+\frac{32\pi}{5}-8\pi$

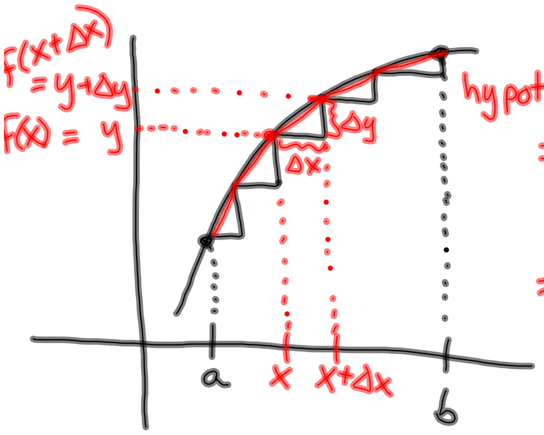
$=\ldots$

(c) about $x=4$

$\int_0^2 \pi(4-y^2)^2\,dy$

## 6.4 Arc Length & Surfaces of Revolution

The arc length $s$ of a smooth curve $f$ from $a$ to $b$ is $s=\int_a^b\sqrt{1+[f'(x)]^2}\,dx$




hypotenuse has length $\sqrt{(\Delta x)^2+(\Delta y)^2}$

$=\sqrt{(\Delta x)^2\left[1+\left(\frac{\Delta y}{\Delta x}\right)^2\right]}$

$=\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\cdot\Delta x$

MVT $f(x)-f(x+\Delta x)=f'(c)\cdot\Delta x$

$\frac{\Delta y}{\Delta x}=f'(c)$

6. $y=\frac{3}{2}x^{2/3}+4,\ [1,27]$

$$S=\int_1^{27}\sqrt{1+\left[x^{-1/3}\quad\right]^2}\,dx$$

$$=\int_1^{27}\sqrt{1+x^{-2/3}}\,dx=\int_1^{27}\sqrt{\frac{x^{2/3}+1}{x^{2/3}}}\,dx$$

$$=\int_1^{27}\frac{\sqrt{x^{2/3}+1}\,dx}{x^{1/3}}$$

$u=x^{2/3}+1$

$du=\frac{2}{3}x^{-1/3}\,dx$

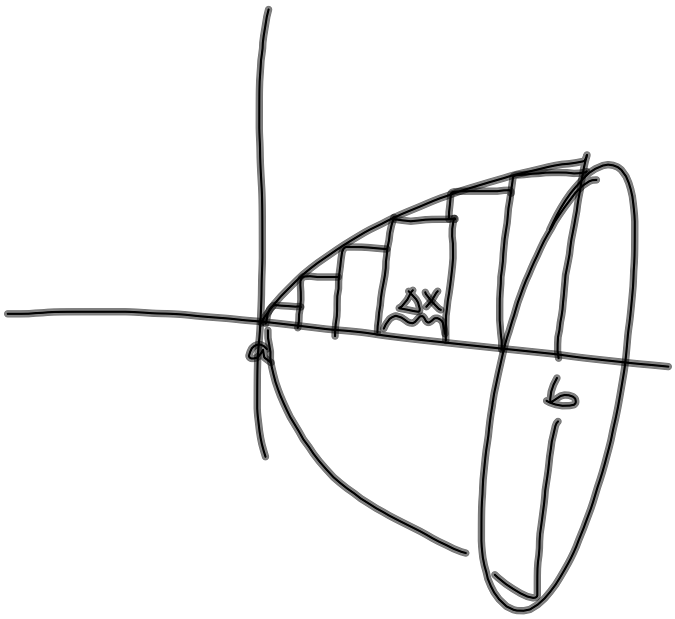
$\frac{3}{2}du=\frac{dx}{x^{1/3}}$

$$=\int_{x=1}^{27}\frac{3}{2}u^{1/2}du$$

$$=u^{3/2}=\left(x^{2/3}+1\right)^{3/2}\Big|_1^{27}=10-2=\boxed{8}$$

18. $y=\ln x\quad,\ [1,5]$

$$\int_1^5\sqrt{1+\left(\frac{1}{x}\right)^2}\,dx$$

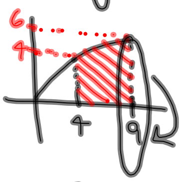



$$V = \int_a^b \pi r^2 \cdot \underset{\overset{\downarrow}{h}}{\Delta x}$$

# Area of a Surface of Revolution

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, dx$$

34. $y = 2\sqrt{x}$ , $[4,9]$ revolve about x-axis $r(x) = 2\sqrt{x}$ ; $f'(x) = \frac{1}{\sqrt{x}}$




$$\int_4^9 2\pi(2\sqrt{x})\sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2}\, dx$$

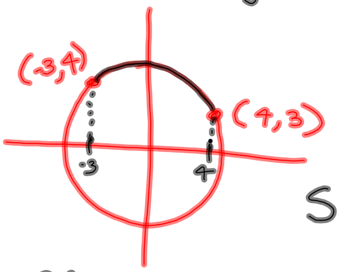
$$= \int_4^9 4\pi\sqrt{x} \cdot \sqrt{1 + \frac{1}{x}}\, dx = \int_4^9 4\pi\cancel{\sqrt{x}}\sqrt{\frac{x+1}{\cancel{x}}}\, dx$$

$$= \int_4^9 4\pi\sqrt{x+1}\, dx = \int_{x=4}^{9} 4\pi\, u^{1/2} du = \frac{8\pi}{3} u^{3/2} =$$

$u = x+1$
$du = dx$

$$= \frac{8\pi}{3}(x+1)^{3/2}\Big|_4^9 = \frac{8\pi}{3}(10)^{3/2} - \frac{8\pi}{3}(5)^{3/2}$$

32. Find arc length from $(-3,4)$ clockwise to $(4,3)$ along the circle $x^2+y^2=25$.




top of circle: $y=\sqrt{25-x^2}$

$$y' = \frac{1}{2}(25-x^2)^{-1/2}\cdot(-2x)$$

$$S=\int_{-3}^{4}\sqrt{1+\left(\frac{-x}{\sqrt{25-x^2}}\right)^2}\,dx$$

$$=\int_{-3}^{4}\sqrt{1+\frac{x^2}{25-x^2}}\,dx=\int_{-3}^{4}\sqrt{\frac{25}{25-x^2}}\,dx$$

$$=\int_{-3}^{4}\frac{5\,dx}{\sqrt{25-x^2}}$$

$$\int\frac{du}{\sqrt{a^2-u^2}}=\arcsin\frac{u}{a}+c$$

$$=5\text{arcsn}\frac{5}{x}\Big|_{-3}^{4}\ \ldots$$

6.4 # 5, 7, 13, 33, 35