6.4

35. $y=\frac{x^3}{6}+\frac{1}{2x}$, $[1,2]$

area of surface
obtained by
revolving around
x-axis

$$\int_1^2 2\pi\left(\frac{x^3}{6}+\frac{1}{2x}\right)\cdot\sqrt{1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2}\,dx$$

$$=2\pi\int_1^2 \frac{x^4+3}{6x}\sqrt{1+\left(\frac{x^4-1}{2x^2}\right)^2}\,dx$$

$$=2\pi\int_1^2 \frac{x^4+3}{6x}\sqrt{\frac{4x^4+x^8-2x^4+1}{4x^4}}\,dx$$

$$=2\pi\int_1^2 \frac{x^4+3}{6x}\cdot\frac{\sqrt{x^8+2x^4+1}}{2x^2}\,dx$$

$$=2\pi\int_1^2 \frac{x^4+3}{12x^3}\sqrt{(x^4+1)^2}\,dx$$

$$=\frac{\pi}{6}\int_1^2 \frac{(x^4+3)(x^4+1)\,dx}{x^3}$$

$$=\frac{\pi}{6}\int_1^2 \frac{x^8+4x^4+3}{x^3}dx=\frac{\pi}{6}\int_1^2(x^5+4x+3x^{-3})$$

7. $y=\frac{x^4}{8}+\frac{1}{4x^2}$ $[1,2]$

$$\left(\frac{1}{4}x^{-2}\right)'$$

$$-\frac{1}{2}x^{-3}$$

$$\int_1^2\sqrt{1+\left(\frac{x^3}{2}-\frac{1}{2x^3}\right)^2}\,dx$$

$$=\int_1^2\sqrt{1+\left(\frac{x^6-1}{2x^3}\right)^2}\,dx$$

$$=\int_1^2\sqrt{1+\frac{x^{12}-2x^6+1}{4x^6}}=\int_1^2\sqrt{\frac{4x^6+x^{12}-2x^6+1}{4x^6}}\,dx$$

$$=\int_1^2\frac{\sqrt{x^{12}+2x^6+1}}{2x^3}dx=\int_1^2\frac{\sqrt{(x^6+1)^2}}{2x^3}dx$$

$$=\int_1^2\left(\frac{x^6}{2x^3}+\frac{1}{2x^3}\right)dx=\ldots$$

7.1 Basic Integration Rules

$$\int \frac{4}{x^2+9}\,dx$$

$$= \boxed{\frac{4}{3}\arctan\frac{x}{3}+C}$$

$$\cancel{\frac{ab}{c} = \frac{a}{c}\cdot\frac{b}{c}}$$

$$\int \frac{4x}{x^2+9}\,dx$$

$u = x^2+9$

$du = 2x\,dx$

$2du = 4x\,dx$

$$\int \frac{2du}{u} = 2\ln|u|$$

$$= \boxed{2\ln(x^2+9)+C}$$

$$\int \frac{4x^2}{x^2+9}\,dx$$

$$\begin{array}{r|l} & 4 + \frac{-36}{x^2+9} \\ \hline x^2+9 & 4x^2 \\ & -(4x^2+36) \\ \hline & -36 \end{array}$$

$$= \int \left(4 + \frac{-36}{x^2+9}\right) dx$$

$$= \boxed{4x - 12\arctan\left(\frac{x}{3}\right) + C}$$

$$\int \frac{1}{1+e^x}\,dx = \int \frac{1+e^x-e^x}{1+e^x}\,dx$$

$u = 1+e^x$

$du = e^x\,dx$

$$= \int \frac{1+e^x}{1+e^x}\,dx - \int \frac{e^x\,dx}{1+e^x}$$

$$= \boxed{x - \ln(1+e^x) + C}$$

$$\int \tan^2 2x \, dx$$

$$= \int (\sec^2 2x - 1) \, dx$$

$$= \int \sec^2 2x \, dx - \int dx$$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$$= \boxed{\frac{1}{2}\tan 2x - x + C}$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

$$\int \cot x \ln(\sin x) \, dx = \int u \, du = \frac{u^2}{2} + c$$

$u = \ln(\sin x)$

$du = \frac{1}{\sin x} \cdot \cos x \, dx$

$du = \cot x \, dx$

$$= \boxed{\frac{(\ln(\sin x))^2}{2} + c}$$

$\boxed{7.1 \ \#5\text{-}53 \text{ odd}}$