

answer to bonus B:

$$-\frac{\pi}{2} + \frac{2\pi}{3} - 2\sqrt{3} + 4 \approx 1.059$$

Bonus A

$$\int \cos^3 x dx = \int (1 - \sin^2 x) \cos x dx$$

$$u = \sin x$$

$$du = \cos x dx$$

$$= \int (1 - u^2) du = u - \frac{1}{3} u^3 + C$$

$$= \boxed{\sin x - \frac{1}{3} \sin^3 x + C}$$

Bonus C

$$S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt \quad (0, 3)$$

$$S'(x) = \sin\left(\frac{\pi x^2}{2}\right) = 0$$

$$\frac{\pi x^2}{2} = 0 + \pi k, \quad k \in \mathbb{Z}$$

$$\pi x^2 = 2\pi k$$

$$x^2 = 2k$$

$$x = \sqrt{2k} = \boxed{\sqrt{2}, 2, \sqrt{6}, 2\sqrt{2}}$$

## 7.2 Integration by Parts

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$$\frac{d}{dx}[uv] = u \cdot \frac{d}{dx}[v] + v \cdot \frac{d}{dx}[u]$$

$$= uv' + vu'$$

Integrating both sides w.r.t.  $x$  yields:

$$uv = \int uv' dx + \int vu' dx$$

$$uv = \int u dv + \int v du$$

Rearranging yields:

$$\int u dv = uv - \int v du$$

$$\int x e^x dx$$

$$\int u dv = uv - \int v du$$

$$u = x \quad dv = e^x dx$$

$$du = dx \quad \int dv = \int e^x dx$$

$$v = e^x$$

$$\int x e^x dx = x e^x - \int e^x dx$$

$$= x e^x - e^x + c$$

$$6. \int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \int \frac{1}{2} e^{2x} \cdot 2x dx$$

$$u = x^2 \quad dv = e^{2x} dx$$

$$du = 2x dx \quad v = \frac{1}{2} e^{2x}$$

$$u = x \quad dv = e^{2x} dx$$

$$du = dx \quad v = \frac{1}{2} e^{2x}$$

$$= \frac{1}{2} x^2 e^{2x} - \left( \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} dx \right)$$

$$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C$$

$$16. \int x^4 \ln x dx = \frac{1}{5} x^5 \ln x - \int \frac{1}{5} x^5 \cdot \frac{1}{x} dx$$

$$u = \ln x \quad dv = x^4 dx$$

$$du = \frac{1}{x} dx \quad v = \frac{1}{5} x^5$$

$$= \frac{1}{5} x^5 \ln x - \frac{1}{5} \int x^4 dx$$

$$= \frac{1}{5} x^5 \ln x - \frac{1}{25} x^5 + C$$

$$34. \int 4 \arccos x \, dx = 4x \arccos x + \int \frac{+4x}{\sqrt{1-x^2}} \, dx$$

$$u = \arccos x \quad dv = 4 \, dx$$

$$du = \frac{-1}{\sqrt{1-x^2}} \, dx \quad v = 4x$$

$$w = 1-x^2$$

$$dw = -2x \, dx$$

$$-2 \, dw = 4x \, dx$$

$$= 4x \arccos x + \int \frac{-2 \, dw}{\sqrt{w}}$$

$$= 4x \arccos x + \int -2w^{-1/2} \, dw$$

$$= 4x \arccos x - 4w^{1/2}$$

$$= \boxed{4x \arccos x - 4\sqrt{1-x^2} + C}$$

$$30. \int x^2 \cos x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x$$

$$u = x^2 \quad dv = \cos x \, dx \quad ? \quad +C$$

$$du = 2x \, dx \quad v = \sin x$$

$$x^2 \sin x - \int 2x \sin x \, dx$$

$$u = 2x \quad dv = \sin x \, dx$$

$$du = 2 \, dx \quad v = -\cos x$$

$$x^2 \sin x - (-2x \cos x - \int -2 \cos x \, dx)$$

$$\boxed{x^2 \sin x + 2x \cos x - 2 \sin x + C}$$

$$36. \int e^x \cos 2x dx$$

$$u = \cos 2x \quad dv = e^x dx$$

$$du = -2 \sin 2x dx \quad v = e^x$$

$$\int e^x \cos 2x dx = e^x \cos 2x - \int -2 e^x \sin 2x dx$$

HW:  
7.2 #1-35 odd

$$= e^x \cos 2x + 2 \int e^x \sin 2x dx$$

$$u = \sin 2x \quad dv = e^x dx$$

$$du = 2 \cos 2x dx \quad v = e^x$$

$$\int e^x \cos 2x dx = e^x \cos 2x + 2 \left( e^x \sin 2x - \int 2 e^x \cos 2x dx \right)$$

$$\int e^x \cos 2x dx = e^x \cos 2x + 2 e^x \sin 2x - 4 \int e^x \cos 2x dx$$

$$5 \int e^x \cos 2x dx = e^x \cos 2x + 2 e^x \sin 2x$$

$$\int e^x \cos 2x dx = \boxed{\frac{1}{5} e^x \cos 2x + \frac{2}{5} e^x \sin 2x + C}$$