7.1

37. $\int \frac{2}{e^{-x}+1}dx = \int \frac{2\,dx}{\frac{1}{e^x}+\frac{e^x}{e^x}}$

$= \int \frac{2e^x\,dx}{1+e^x} = \int \frac{2\,du}{u} = 2\ln|u| + c$

$u = 1+e^x$
$du = e^x dx$
$2du = 2e^x dx$

$= 2\ln(1+e^x) + c$

49. $\int \frac{\tan \frac{2}{t}}{t^2}dt = \int -\frac{1}{2}\tan u\,du$

$u = \frac{2}{t}$
$du = -\frac{2}{t^2}dt$
$-\frac{1}{2}du = \frac{dt}{t^2}$

$= -\frac{1}{2}\left[-\ln|\cos u|\right] + c$

$= \frac{1}{2}\ln\left|\cos\frac{2}{t}\right| + c$

43. $\int \frac{1}{\cos\theta - 1}\, d\theta =$

$= \int \frac{1}{\cos\theta - 1} \cdot \frac{\cos\theta + 1}{\cos\theta + 1}\, d\theta =$

$= \int \frac{\cos\theta + 1}{\cos^2\theta - 1}\, d\theta = -\int \frac{\cos\theta + 1}{\sin^2\theta}\, d\theta$

$= -\int \frac{\cos\theta\, d\theta}{\sin^2\theta} - \int \frac{d\theta}{\sin^2\theta}$

$= -\int \csc\theta \cot\theta\, d\theta - \int \csc^2\theta\, d\theta$

$= \boxed{\csc\theta + \cot\theta + C}$

51. $\int \frac{3\,dx}{\sqrt{6x - x^2}} = \int \frac{3\,dx}{\sqrt{-(x^2 - 6x + 9) + 9}}$

$= \int \frac{3\,dx}{\sqrt{3^2 - (x-3)^2}} = \boxed{3 \arcsin \frac{x-3}{3} + C}$

$a = 3$

$u = x - 3$

$du = dx$

$\int \frac{du}{\sqrt{a^2 - u^2}}$

$= \arcsin \frac{u}{a}$

7.2

$$\int u\,dv = uv - \int v\,du$$

8. $\int \ln 3x\,dx = x\ln 3x - \int x \cdot \frac{3}{x}\,dx$

$u = \ln 3x \quad dv = dx$

$du = \frac{3}{x}dx \quad v = x$

$= x\ln 3x - 3x + C$

14. $\int \frac{e^{1/t}}{t^2}\,dt = -\int e^u\,du = -e^{1/t} + C$

$u = 1/t$

$du = -\frac{1}{t^2}dt$

48. $\int_0^1 x^2 e^x dx = x^2e^x - \int 2xe^x dx$

$u = x^2 \quad dv = e^x dx$

$du = 2x dx \quad v = e^x$

$u = 2x \quad dv = e^x dx$

$du = 2 dx \quad v = e^x$

$= x^2e^x - \left[2xe^x - \int 2e^x dx\right] =$

$= x^2e^x - 2xe^x + 2e^x \Big|_0^1$

$= [e - 2e + 2e] - 2 = \boxed{e-2}$

28. $\int x \sin x \, dx = -x\cos x + \int \cos x \, dx$

$u = x \quad dv = \sin x \, dx$

$du = dx \quad v = -\cos x$

$= \boxed{-x\cos x + \sin x + C}$

67. $\int \frac{\ln x}{x} dx$

$u = \ln x$

$du = \frac{dx}{x}$

$= \int u\,du$

$= \frac{u^2}{2}$

$= \frac{(\ln x)^2}{2} + C$

68. $\int x \ln x \, dx$

$u = \ln x \quad dv = x\,dx$

$du = \frac{dx}{x} \quad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \cdot \frac{dx}{x}$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

58. $\int_0^{\pi/4} x \sec^2 x \, dx = x\tan x - \int \tan x \, dx$

$u = x \quad dv = \sec^2 x\,dx$

$du = dx \quad v = \tan x$

$= x\tan x + \ln|\cos x| \Big|_0^{\pi/4}$

$\log_a(b^p)$

$= p\log_a b$

$\ln 2^{-1/2} = -\frac{1}{2}\ln 2$

$= \frac{\pi}{4} + \ln\left(\frac{1}{\sqrt{2}}\right) - [\ln 1]$, where $\ln 1 = 0$

$= \frac{\pi}{4} - \frac{1}{2}\ln 2$

Solve the differential equation.

40. $\frac{dy}{dx} = x^2\sqrt{x-1}$

$\int dy = \int x^2\sqrt{x-1}\,dx$ separation of variables

$y = \int x^2\sqrt{x-1}\,dx = \int (u+1)^2 u^{1/2}\,du$

$u = x-1$ $\quad x = u+1$

$du = dx$ $\quad x^2 = (u+1)^2$

$= \int (u^2+2u+1)\,u^{1/2}\,du$

$= \int \left(u^{5/2} + 2u^{3/2} + u^{1/2}\right)du$

$= \frac{2}{7}u^{7/2} + \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$

$y = \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$