$\int u\,dv = vu - \int v\,du$

7.2

21. $\int \frac{xe^{2x}}{(2x+1)^2}dx = \int \frac{xe^{2x}}{4x^2+4x+1}\,dx$

$u = 2x+1 \quad 2x = u-1$

$\frac{du}{2} = \frac{\cancel{2}dx}{\cancel{2}} \quad x = \frac{u-1}{2}$

$u = 2x$

$du = 2dx \quad \frac{1}{2}\int \frac{\frac{u}{2}e^{u}}{(u+1)^2}du$

$= \int \frac{\frac{u-1}{2}\cdot e^{u-1}}{u^2}\cdot\frac{du}{2} = \frac{1}{4}\int\frac{(u-1)e^{u-1}}{u^2}du$

$= \frac{1}{4}\int\frac{\cancel{u}e^{u-1}}{u^{\cancel{2}}}du - \frac{1}{4}\int\frac{e^{u-1}}{u^2}du$

$\frac{1}{2}\int\frac{e^{2x}}{2x+1}dx$

$u=$

Ans? $\frac{e^{2x}}{4(2x+1)} + C$

7.2

21. $\int \frac{xe^{2x}}{(2x+1)^2}dx$

$dv = e^{2x}dx \qquad u = \frac{x}{(2x+1)^2}$

$v = \frac{1}{2}e^{2x}$

$du = \frac{(2x+1)^2 - x[2(2x+1)^1\cdot 2]}{(2x+1)^4}dx$

$= \frac{2x+1-4x}{(2x+1)^3}dx$

$du = \frac{1-2x}{(2x+1)^3}dx$

7.2

Ans? $\frac{e^{2x}}{4(2x+1)}+C$

21. $\int \frac{xe^{2x}}{(2x+1)^2}dx$

$= \int \frac{e^{2x}dx}{x^{-1}(4x^2+4x+1)} = \int \frac{e^{2x}dx}{4x+4+\frac{1}{x}}$

$u=e^{2x}$ $dv=\frac{xdx}{(2x+1)^2}$

$du=2e^{2x}$

$v=\int \frac{xdx}{(2x+1)^2} = \int \frac{\frac{a-1}{2}\cdot\frac{da}{2}}{a^2} = \frac{1}{4}\int\left(\frac{1}{a}-\frac{1}{a^2}\right)$

$a=2x+1$ $x=\frac{a-1}{2}$

$da=2dx$

$=\frac{1}{4}\left(\ln|a|-\frac{1}{a}\right)$

$=\frac{1}{4}\left(\ln|2x+1|-\frac{1}{2x+1}\right)$

$=\frac{1}{4}e^{2x}\left(\ln|2x+1|-\frac{1}{2x+1}\right)$

$-\int v\,du$ ← ew!

7.3 Trigonometric Integrals

$\sin^2x+\cos^2x=1$

$\tan^2x+1=\sec^2x$

$\cot^2x+1=\csc^2x$

$\sin 2x=2\sin x\cos x$

$\cos 2x=2\cos^2x-1$

$=1-2\sin^2x$

4. $\int \cos^3x\sin^4x\,dx = \int(1-\sin^2x)\cos x\cdot\sin^4x\,dx$

$\cos^2x\cdot\cos x$

$(1-\sin^2x)\cos x$

$=\int\sin^4x\cos x\,dx - \int\sin^6x\cos x\,dx$

$u=\sin x$ $u=\sin x$

$du=\cos x\,dx$ $du=\cos x\,dx$

$\int u^4du$ $-\int u^6du$

$\frac{1}{5}u^5$ $-\frac{1}{7}u^7$

$\boxed{\frac{1}{5}\sin^5x - \frac{1}{7}\sin^7x+C}$

12. $\int \sin^2 2x \, dx = \int (2\sin x \cos x)^2 \, dx$

$\cancel{= \int (1 - \cos^2 2x) \, dx}$

$= \int 4 \sin^2 x \cos^2 x \, dx$

$= \int 4 \sin^2 x \left(\frac{\cos 2x + 1}{2}\right) dx$

$= \int 4 \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right) dx$

$= \int (1 - \cos^2 2x) \, dx$

$\cos 2x = 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

HW

read 7.3 examples for understanding