7.3 # 12

$$\int \sin^2 2x\,dx = \int (1-\cos^2 2x)\,dx = \int 1\,dx - \int \frac{\cos 4x + 1}{2}\,dx =$$

$$= x - \frac{1}{2}\int \cos 4x\,dx - \frac{1}{2}\int dx = x - \frac{1}{2}x - \frac{1}{2}\left(\frac{1}{4}\sin 4x\right) + C$$

$$= \boxed{\frac{1}{2}x - \frac{1}{8}\sin 4x + C}$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

7.2 #21

$$\int \frac{xe^{2x}\,dx}{(2x+1)^2} = \frac{1}{2}\int \frac{\overbrace{2xe^{2x} + e^{2x}}\,\overbrace{-e^{2x}}}{(2x+1)^2}\,dx =$$

$$= \frac{1}{2}\int \frac{(2x+1)e^{2x}}{(2x+1)^2}\,dx - \frac{1}{2}\int \frac{e^{2x}}{(2x+1)^2}\,dx$$

$$= \frac{1}{2}\int \frac{e^{2x}\,dx}{2x+1} - \frac{1}{2}\int \frac{e^{2x}}{(2x+1)^2}\,dx$$

$$u = \frac{1}{2x+1} \quad dv = e^{2x}\,dx$$

$$du = \frac{-2\,dx}{(2x+1)^2} \quad v = \frac{1}{2}e^{2x}$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{2(2x+1)} + \int \left(\frac{1}{\cancel{2}}e^{2x}\right)\left(\frac{+\cancel{2}\,dx}{(2x+1)^2}\right)\right] - \frac{1}{2}\int \frac{e^{2x}}{(2x+1)^2}\,dx$$

$$= \frac{e^{2x}}{4(2x+1)} + \frac{1}{2}\int \frac{e^{2x}\,dx}{(2x+1)^2} - \frac{1}{2}\int \frac{e^{2x}\,dx}{(2x+1)^2} + C$$

$$= \boxed{\frac{e^{2x}}{4(2x+1)} + C}$$

7.3

26. $\int \tan^2 x \, dx$

$$\int (\sec^2 x - 1)\, dx$$

$$\boxed{= \tan x - x + C}$$

38. $\int \frac{\tan^2 x}{\sec^5 x} dx$

$$= \int \frac{\sin^2 x}{\cos^{\cancel{2}} x} \cdot \frac{\cos^{\cancel{5}\,3} x}{1}$$

$$= \int (\sin^2 x \cdot \cos^2 x \cdot \cos x)\, dx$$

$$= \int \sin^2 x\,(1 - \sin^2 x)(\cos x)\, dx = \int u^2 (1 - u^2)\, du$$

$u = \sin x$

$du = \cos x\, dx$

$$= \int u^2 - u^4\, du$$

$$\frac{1}{3}u^3 - \frac{1}{5}u^5 = \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$$

7.2 #19

$$\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \boxed{\frac{u^3}{3}} + C$$

$$= \frac{(\ln x)^3}{3} + C$$

7.3

16. $\int x^2 \sin^2 x\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$u = x^2 \qquad v = \int \sin^2 x\,dx$

$du = 2x\,dx \qquad dv = \sin^2 x\,dx$

$-\left(\frac{\cos 2x - 1}{2}\right) = \sin^2 x$

$\int \frac{1-\cos 2x}{2}\,dx$

$\int \frac{dx}{2} - \int \frac{\cos 2x}{2}\,dx$

$\frac{1}{2}x - \frac{1}{4}\sin 2x = V$

$x^2\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right) - \int 2x\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right)dx$

$\frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \int\left(x^2 - \frac{1}{2}x\sin 2x\right)dx$

$\frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \left[\int x^2\,dx - \int\left(\frac{1}{2}x\sin 2x\right)dx\right]$

$\frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \left[\frac{1}{3}x^3 - \int\left(\frac{1}{2}x\sin 2x\right)dx\right]$

$\frac{1}{2}\int (x\sin 2x)\,d$

$w = x \qquad z = -\frac{1}{2}\cos 2$

$dw = dx \quad dz = \sin 2x\,d$

$\frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{3}x^3 + \frac{1}{2}\left(-\frac{1}{2}x\cos 2x - \int -\frac{1}{2}\cos 2x\,dx\right)$

$\frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{3}x^3 + \frac{1}{2}\left(-\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x\,dx\right)$

$= \frac{1}{2}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{3}x^3 - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$

7.3

#24