6. $\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$

$u = e^x \quad dv = \cos x\,dx$

$du = e^x dx \quad v = \sin x$

$u = e^x \quad dv = \sin x\,dx$

$du = e^x dx \quad v = -\cos x$

$= e^x \sin x - \left(-e^x \cos x + \int e^x \cos x\,dx\right)$

$= e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$

$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$

$\int e^x \cos x = \boxed{\dfrac{e^x \sin x + e^x \cos x}{2} + C}$

8. $\int \sin^2 x \cos^2 x\,dx =$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)dx$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\frac{1}{4}\int (1-\cos^2 2x)\,dx$

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

$\frac{1}{4}\int dx - \frac{1}{4}\int \cos^2 2x\,dx$

$\frac{1}{4}x - \frac{1}{4}\int \frac{1+\cos 4x}{2}dx = \frac{1}{4}x - \frac{1}{8}\int dx - \frac{1}{8}\int \cos 4x\,dx$

$= \frac{1}{4}x - \frac{1}{8}x - \frac{1}{8}\left(\frac{\sin 4x}{4}\right) + C$

$= \boxed{\frac{1}{8}x - \frac{\sin 4x}{32} + C}$

9. $\int \tan^3 x \sec^3 x \, dx$

$\int \underbrace{\tan^2 x}_{\sec^2 x - 1} \sec^2 x \underbrace{\sec x \tan x \, dx}$

$\int (\sec^4 x - \sec^2 x) \sec x \tan x$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\int (u^4 - u^2) \, du$

10. $\int \sin^3 x \cos^3 x \, dx$

$= \int \sin^2 x \sin x \cos^3 x \, dx$

$= \int (1 - \cos^2 x) \cos^3 x \cdot \sin x \, dx$

$= \int (\cos^3 x - \cos^5 x) \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$= -\int (u^3 - u^5) \, du$

11. $\int \frac{1}{(25-x^2)^{3/2}}\,dx = \int \frac{5\cos\theta\, d\theta}{(25-(5\sin\theta)^2)^{3/2}}$

$x = 5\sin\theta$

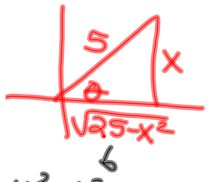
$dx = 5\cos\theta\, d\theta$

$\frac{x}{5} = \sin\theta$

$\theta = \sin^{-1}\frac{x}{5}$

$= \int \frac{5\cos\theta\, d\theta}{(25-25\sin^2\theta)^{3/2}}$

$= \int \frac{5\cos\theta\, d\theta}{\left(\sqrt{25(\underbrace{1-\sin^2\theta}_{\cos^2\theta})}\right)^3}$



$x^2 + b^2 = 5^2$

$b^2 = 25 - x^2$

$b = \sqrt{25-x^2}$

$= \int \frac{5\cos\theta\, d\theta}{(5\cos\theta)^3}$

$= \int \frac{d\theta}{25\cos^2\theta} = \frac{1}{25}\int \sec^2\theta\, d\theta$

$= \frac{1}{25}\tan\theta + C$

$= \frac{1}{25} \cdot \frac{x}{\sqrt{25-x^2}} + C$

Area Between Curves

$f(x) = \sin x + 5$ $\quad 0 \le x \le 5\pi$

$g(x) = -\sin x + 5$

1. find where f & g intersect on the interval.

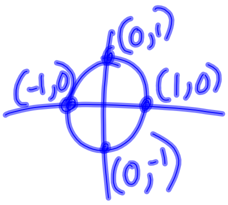
$f(x) = g(x)$

$\sin x + 5 = -\sin x + 5$

$2\sin x = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$



$5\int_0^{\pi} (\sin x + 5 - (-\sin x + 5))\,dx$

$= 5\int_0^{\pi} 2\sin x\, dx = 10\int_0^{\pi} \sin x\, dx =$

$= -10\cos x \Big|_0^{\pi} = -10(-1-1) = -10(-2)$

$= \boxed{20}$

Volume of solid of revolution

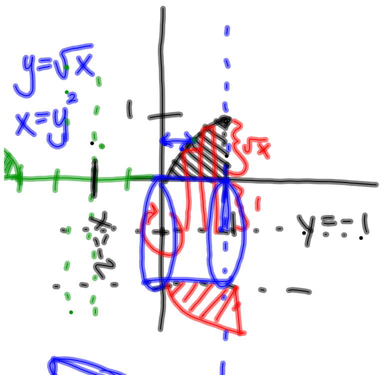
$f(x) = \sqrt{x}$

$y = 0$

$x = 1$

a. rot. @ x-axis

$$\int_0^1 \pi(\sqrt{x})^2\,dx = \int_0^1 \pi x\,dx = \frac{\pi x^2}{2}\Big|_0^1 = \boxed{\frac{\pi}{2}}$$



b. rot @ y=-1

$$\int_0^1 \pi(\sqrt{x}+1)^2\,dx - \pi(1)^2\cdot 1$$

c. rot @ x=1

$$\int_0^1 \pi(1-y^2)^2\,dy$$

d. @ x=-2

$$\pi(3)^2\cdot 1 - \int_0^1 \pi(2+y^2)^2\,dy$$