Volume

$y=x^2+1$, $y=-x^2+2x+5$, $x=0$, $x=3$

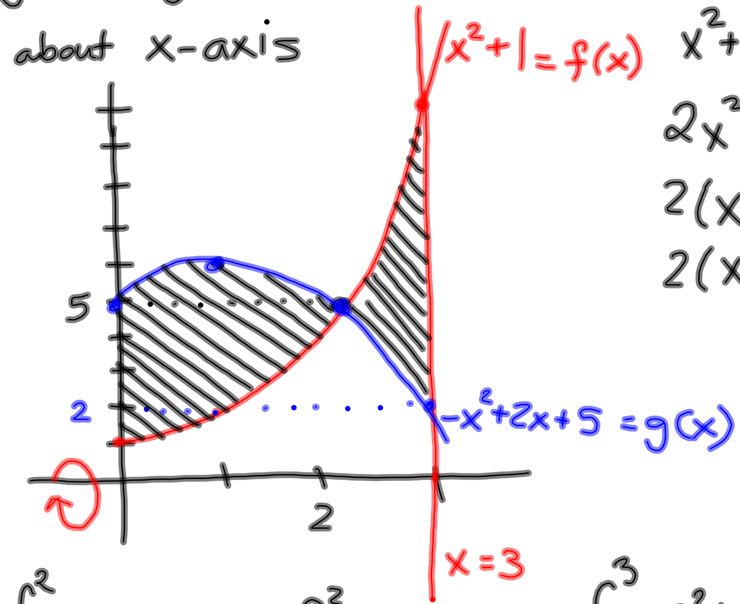
about x-axis

$x^2+1=-x^2+2x+5$

$2x^2-2x-4=0$

$2(x^2-x-2)=0$

$2(x-2)(x+1)=0$



$$\int_0^2 \pi g^2(x)\,dx - \int_0^2 \pi f^2(x)\,dx + \int_2^3 \pi f^2(x)\,dx - \int_2^3 \pi g^2(x)\,dx$$

Volume of solid of revolution

$f(x)=\sqrt{x}$

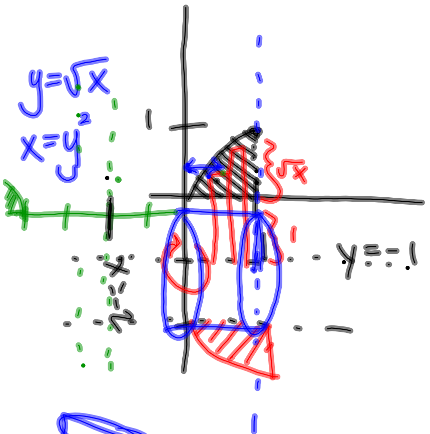
$y=0$

$x=1$

a. rot. @ x-axis

$$\int_0^1 \pi(\sqrt{x})^2\,dx = \int_0^1 \pi x\,dx = $$

$$= \frac{\pi x^2}{2}\Big|_0^1 = \boxed{\frac{\pi}{2}}$$



b. rot @ $y=-1$

$$\int_0^1 \pi(\sqrt{x}+1)^2\,dx - \pi(1)^2\cdot 1$$

c. rot @ $x=1$

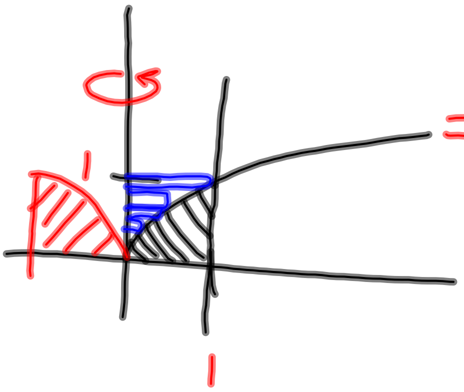
$$\int_0^1 \pi(1-y^2)^2\,dy$$

d. @ $x=-2$

$$\pi(3)^2\cdot 1 - \int_0^1 \pi(2+y^2)^2\,dy$$

$y = \sqrt{x}$, $x=0$, $x=1$ @ y-axis

$y^2 = x$



outer cylinder − inner space

$$= \pi(1)^2 \cdot 1 - \int_0^1 \pi (y^2)^2\,dy$$

$$= \pi - \frac{\pi}{5} y^5 \Big|_0^1$$

$$= \boxed{\frac{4\pi}{5}}$$

6.4 #6 Arc Length $= \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

$y = \frac{3}{2}x^{2/3} + 4$, $[1, 27]$

$y' = x^{-1/3} = \frac{1}{\sqrt[3]{x}}$

$$\int_1^{27} \sqrt{1+\left[\frac{1}{\sqrt[3]{x}}\right]^2}\,dx = \int_1^{27}\sqrt{\frac{x^{2/3}+1}{x^{2/3}}}\,dx$$

$$= \int_1^{27}\sqrt{x^{2/3}+1}\cdot\frac{dx}{x^{1/3}}$$

$u = x^{2/3}+1$

$du = \frac{2}{3}x^{-1/3}\,dx$

$\frac{3}{2}du = x^{-1/3}\,dx$

$$= \frac{3}{2}\int_{x=1}^{27} u^{1/2}\,du = \frac{3}{2}\cdot\frac{2}{3} u^{3/2}\Big|_{x=1}^{27} = \left(x^{2/3}+1\right)^{3/2}\Big|_1^{27} =$$

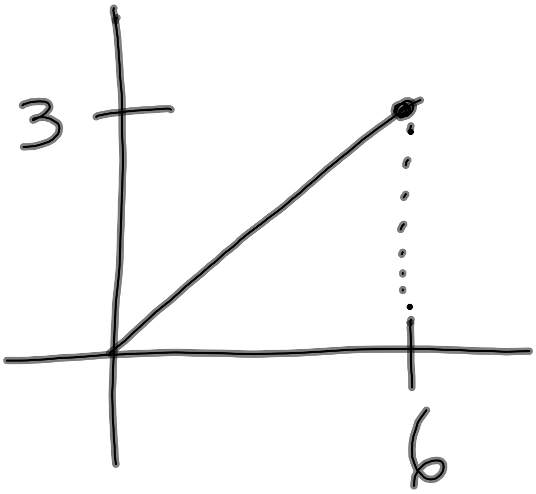
$$= \boxed{10^{3/2} - 2^{3/2}} \approx$$

Surface of Revolution

$2\pi r h$, $h=$ arc length

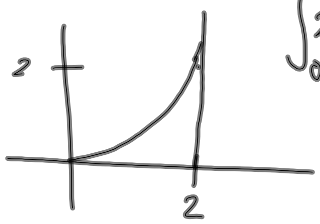
6.4

36. $y=\frac{x}{2}$, $[0,6]$ about x-axis



$$\int_0^6 2\pi\left(\frac{x}{2}\right)\cdot\sqrt{1+\left(\frac{1}{2}\right)^2}\,dx$$

$y=\frac{x^2}{2}$, $[0,2]$ @ x-axis



$$\int_0^2 2\pi\left(\frac{x^2}{2}\right)\sqrt{1+x^2}\,dx$$

surface area $= \int_a^b 2\pi r h$, $r=$ dist. from func. to axis of rot., $h=$ arc length

$$\pi\int_0^2 x^2\sqrt{1+x^2}\,dx$$

$u=1+x^2$, $x^2=u-1$

$du=2x\,dx$

Let $x=\tan\theta$

$x^2=\tan^2\theta$

$dx=\sec^2\theta\,d\theta$

$$\pi\int_{x=0}^{2}\tan^2\theta\underbrace{\sqrt{1+\tan^2\theta}}_{\sqrt{\sec^2\theta}}\cdot\sec^2\theta\,d\theta$$

$$=\pi\int_{x=0}^{2}\tan^2\theta\sec\theta\sec^2\theta\,d\theta$$

$$=\pi\int_{x=0}^{2}\frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{1}{\cos^3\theta}\,d\theta=\pi\int_{x=0}^{2}\frac{\sin^2\theta}{\cos^5\theta}\,d\theta\ \ldots$$

arc length
surf area
volume
trig double ∠'s
pyth id's

inverse trig fn's
$\int$ csc, sec, tan, cot