

7.5 Partial Fractions

$$\int \frac{1}{x^2-5x+6} dx$$

$$\frac{1}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2}$$

$(x-3)(x-2)$

$$\rightarrow \frac{A(x-2) + B(x-3)}{(x-3)(x-2)} = \frac{Ax - 2A + Bx - 3B}{x^2 - 5x + 6}$$

$$= \frac{(A+B)x + (-2A-3B)}{x^2-5x+6}$$

$$A+B=0 \quad A=-B$$

$$-2A-3B=1$$

$$-2(-B)-3B=1 \quad B=\frac{-1}{A}=1$$

$$\int \left(\frac{1}{x-3} + \frac{-1}{x-2} \right) dx$$

$$= \ln|x-3| - \ln|x-2| + C$$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x} dx$$

$$\times (x^2+2x+1)$$

$$\times (x+1)(x+1)$$

$$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2}$$

$$= \frac{Ax^2 + 2Ax + A + Bx^2 + Bx + Cx}{x(x+1)^2}$$

$$= \frac{(A+B)x^2 + (2A+B+C)x + A}{x(x+1)^2}$$

$$5 = A+B$$

$$5 = 6+B$$

$$20 = 2(6) + (-1) + C$$

$$20 = 2A+B+C$$

$$-1 = B$$

$$C = 20 - 12 + 1$$

$$6 = A$$

$$C = 9$$

$$\int \frac{5x^2+20x+6}{x(x+1)^2} dx = \int \left(\frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2} \right) dx$$

$$= 6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C$$

$$\begin{aligned} u &= x+1 \\ du &= dx \\ 9u^{-2} - 9u^{-1} &= -\frac{9}{u} \end{aligned}$$

$$\int \frac{2x^3 - 4x - 8}{(x^2 - x)(x^2 + 4)} dx$$

$$\frac{2x^3 - 4x - 8}{x(x-1)(x^2+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+4}$$

$$\frac{A(x-1)(x^2+4) + Bx(x^2+4) + (Cx+D)(x^2-x)}{(Ax-A)(x^2+4)}$$

$$Ax^3 + 4Ax - Ax^2 - 4A + Bx^3 + 4Bx + Cx^3 - Cx^2 + Dx^2 - Dx$$

$$(A+B+C)x^3 + (-A-C+D)x^2 + (4A+4B-D)x - 4$$

$$\begin{cases} 2 = A+B+C \\ 0 = -A-C+D \\ -4 = 4A+4B-D \\ -8 = -4A \end{cases} \Rightarrow A=2$$

$$\begin{cases} B+C=0 \\ D-C=2 \end{cases} \Rightarrow B+D=2$$

$$4B-D=-12 \Rightarrow \begin{cases} 5B=-10 \\ B=-2 \\ D=4; C=2 \end{cases}$$

$$\int \left(\frac{2}{x} + \frac{-2}{x-1} + \frac{2x+4}{x^2+4} \right) dx$$

$$\int \left(\frac{2}{x} + \frac{-2}{x-1} + \frac{2x}{x^2+4} + \frac{4}{x^2+4} \right) dx$$

$$2 \ln|x| - 2 \ln|x-1| + \ln|x^2+4| + 2 \arctan \frac{x}{2} + C$$

7.5
15-27 odd

$$15. \int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8} dx$$

use long division first

$$17. \int \frac{4x^2 + 2x - 1}{x^3 + x^2} dx$$

$$\frac{4x^2 + 2x - 1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} = \frac{A}{x} \cdot \frac{x(x+1)}{x(x+1)} + \frac{B(x+1)}{x^2(x+1)} + \frac{Cx}{x^2(x+1)}$$

$$Ax(x+1) + B(x+1) + Cx^2$$

$$(A+C)x^2 + (A+B)x + B$$

$$A+C=4, A+B=2, B=-1$$

$$A=3, C=1$$

$$\int \left(\frac{3}{x} + \frac{-1}{x^2} + \frac{1}{x+1} \right) dx$$