7.5

23. $$\int \frac{x^2}{x^4-2x^2-8}\,dx$$

$$\frac{x^2}{(x^2-4)(x^2+2)} = \frac{A}{x-2}+\frac{B}{x+2}+\frac{Cx+D}{x^2+2}$$

$(x-2)(x+2)(x^2+2)$

$$A(x+2)(x^2+2)+B(x-2)(x^2+2)+(Cx+D)(x^2-4)$$

$$Ax^3+2Ax+2Ax^2+4A+Bx^3+2Bx-2Bx^2-4B+Cx^3-4Cx+Dx^2-4D$$

$$(A+B+C)x^3+(2A-2B+D)x^2+(2A+2B-4C)x+4A-4B-4D$$

$$A+B+C=0$$
$$2A-2B+D=1$$
$$2A+2B-4C=0 \quad A+B-2C=0$$
$$4A-4B-4D=0 \quad A-B-D=0$$

$$\begin{array}{r} A+B+C=0 \\ -(A+B-2C=0) \\ \hline 3C=0 \\ C=0 \end{array}$$

$$\begin{cases} A+B=0 \\ 2A-2B+D=1 \\ A-B-D=0 \end{cases} \quad A=-B \quad A=\frac{1}{6}$$

$$2(-B)-2B+D=1$$
$$-B-B-D=0$$

$$\begin{array}{r} -4B+D=1 \\ +(-2B-D=0) \\ \hline -6B=1 \end{array} \quad B=-\frac{1}{6}$$

$$D=-2B$$
$$=-2\left(-\frac{1}{6}\right)=\frac{1}{3} \quad D=\frac{1}{3}$$

$$\int\left(\frac{1}{6}\cdot\frac{1}{x-2}+\frac{-1}{6}\cdot\frac{1}{x+2}+\frac{1}{3}\cdot\frac{1}{x^2+2}\right)dx$$

$$\boxed{\frac{1}{6}\ln|x-2|-\frac{1}{6}\ln|x+2|+\frac{1}{3\sqrt{2}}\arctan\frac{x}{\sqrt{2}}+C}$$

27. $$\int\frac{x^2+5}{x^3-x^2+x+3}\,dx \qquad x^3-x^2+x+3 \longrightarrow (x+1)(\qquad)$$

$(-1)^3-(-1)^2+(-1)+3 \Rightarrow -1$ is a zero of x^3-x^2+x+3

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 1 & 3 \\ & & -1 & 2 & -3 \\ \hline & 1 & -2 & 3 & \boxed{0} \end{array}$$

$$(x+1)(x^2-2x+3)$$

$$\frac{x^2+5}{x^3-x^2+x+3}=\frac{A}{x+1}+\frac{Bx+C}{x^2-2x+3}$$

$$Ax^2-2Ax+3A+Bx^2+Cx+Bx+C$$
$$(A+B)x^2+(-2A+B+C)x+3A+C$$

$$A+B=1 \quad B=1-A \qquad -2A+1-A+5-3A=0$$
$$-2A+B+C=0 \qquad -6A+6=0$$
$$3A+C=5 \quad C=5-3A \qquad -6A=-6$$
$$B=0 \quad A=1$$
$$C=2$$

$$\int\left(\frac{1}{x+1}+\frac{2}{x^2-2x+3}\right)dx$$

$x^2-2x+1+2$
$(x-1)^2+2$

$$\ln|x+1|+\frac{2}{\sqrt{2}}\arctan\frac{x-1}{\sqrt{2}}+C$$

44. $\displaystyle\int \frac{\sec^2 x}{\tan x(\tan x+1)}\,dx = \underbrace{\int \frac{1}{u(u+1)}\,du}_{\text{solve by partial fractions}}$

$u = \tan x$

$du = \sec^2 x\,dx$

$\dfrac{1}{u(u+1)} = \dfrac{A}{u} + \dfrac{B}{u+1}$ $\Rightarrow \displaystyle\int \left(\frac{1}{u} + \frac{-1}{u+1}\right) du$

$A(u+1) + Bu$ $\quad A+B=0$

$(A+B)u + A$ $\quad A=1\ ;\ B=-1$ $\quad = \ln|u| - \ln|u+1|$

$$= \boxed{\ln|\tan x| - \ln|\tan x+1| + C}$$

5.7 Solving Differential Equations by Separation of Variables

$y = 5x^3 - \cos x$

what is the differential of y?

$dy = (15x^2 + \sin x)\,dx$

$y' = \dfrac{dy}{dx} = \dfrac{d}{dx}(y)$

5.7 Ex3 - Find the general solution.

$$(x^2+4)\frac{dy}{dx}=xy$$

$$\frac{\cancel{(x^2+4)}\,dy}{y\cancel{(x^2+4)}}=\frac{x\cancel{y}\,dx}{\cancel{y}(x^2+4)}$$

$$\int\frac{dy}{y}=\int\frac{x\,dx}{x^2+4} \qquad u=x^2+4 \quad \frac{du}{2}=\cancel{2}x\,dx \qquad \int\frac{1}{2}\cdot\frac{du}{u}$$

$$e^{\ln|y|}=e^{\left(\frac{1}{2}\ln|x^2+4|+C_1\right)} \qquad a\log_b c=\log_b c^a$$

$$|y|=e^{\ln\sqrt{x^2+4}+C_1} \qquad x^{m+n}=x^m x^n$$

$$|y|=e^{\ln\sqrt{x^2+4}}e^{C_1} \qquad |x|=2$$

$$|y|=\sqrt{x^2+4}\cdot e^{C_1} \qquad x=\pm 2$$

$$y=\pm\sqrt{x^2+4}\cdot e^{C_1}$$

$$y=\underbrace{\pm e^{C_1}}_{=C}\cdot\sqrt{x^2+4}$$

General solution: $\boxed{y=C\sqrt{x^2+4}}$

Ex 4 Find a particular solution.

$$xy\,dx+e^{-x^2}(y^2-1)\,dy=0 \qquad ; \; y(0)=1$$

$$e^{x^2}\cdot\frac{e^{-x^2}(y^2-1)\,dy}{y}=\frac{-x\cancel{y}\,dx}{\cancel{y}}\cdot e^{x^2}$$

$$\int\frac{y^2-1}{y}\,dy=\int -xe^{x^2}\,dx \qquad u=x^2 \quad -\frac{1}{2}du=\frac{\cancel{2}x\,dx}{-\cancel{2}} \qquad \int-\frac{1}{2}e^u\,du=-\frac{1}{2}e^u$$

$$\int\left(y-\frac{1}{y}\right)dy=-\frac{1}{2}e^{x^2}+C$$

$$\frac{1}{2}y^2-\ln|y|=-\frac{1}{2}e^{x^2}+C$$

$$\frac{1}{2}-\ln 1=-\frac{1}{2}e^0+C$$

$$\frac{1}{2}=-\frac{1}{2}+C \qquad C=1$$

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Particular solution:

$$\boxed{\frac{1}{2}y^2-\ln|y|=-\frac{1}{2}e^{x^2}+1}$$