Review

Determine

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty} \frac{4}{n^2}\sum_{i=1}^{n} i$$

$$= \lim_{n\to\infty} \frac{4}{n^2}\cdot\frac{n(n+1)}{2} = \lim_{n\to\infty} \frac{4n^2+4n}{2n^2+\cancel{n}}$$

$$= \boxed{2}$$

5.7

59. $y(1+x^2)y' - x(1+y^2) = 0$; $y(0)=\sqrt{3}$

$$\frac{y(1+\cancel{x^2})\cdot\frac{dy}{\cancel{dx}}}{(\cancel{1+x^2})(1+y^2)} = \frac{x(\cancel{1+y^2})}{(1+x^2)(\cancel{1+y^2})}\cdot dx$$

$$\int \frac{y}{1+y^2}\,dy = \int \frac{x}{1+x^2}\,dx$$

$u = 1+y^2$

$du = 2y\,dy$

$\frac{du}{2} = y\,dy$

$\int \frac{1}{2}\frac{du}{u} = \frac{1}{2}\ln|u|$

$$\frac{1}{2}\ln|1+y^2| = \frac{1}{2}\ln|1+x^2| + C$$

$$e^{\ln\sqrt{1+y^2}} = e^{(\ln\sqrt{1+x^2}+C_1)}$$

$x^{m+n} = x^m x^n$

$$\sqrt{1+y^2} = e^{\ln\sqrt{1+x^2}}\,e^{C_1}$$

$$\sqrt{1+y^2} = C_2\sqrt{1+x^2}$$

$$\rightarrow 1+y^2 = C_3(1+x^2)$$

$$y^2 = C_3 + C_3x^2 - 1$$

$$y = \pm\sqrt{C_3x^2 + C_4}$$

$$\boxed{y = C_5x + C_6}$$ general solution

WOAH!

overzealous, much?

$y(0)=\sqrt{3}$

$1+(\sqrt{3})^2 = c(1+0^2)$

$1+3 = c$

$4 = c$

$$\boxed{y^2 = 3+4x^2}$$ particular

57. $y(x+1)+y'=0 \quad ; \quad y(-2)=1$

$\frac{dy}{dx}=-y(x+1)$

$\int\frac{dy}{y}=\int-(x+1)\,dx$

$x^{m+n}=x^m x^n$

$x^{m-n}=\frac{x^m}{x^n}$

$e^{\ln|y|}=e^{(-\frac{1}{2}x^2-x+C_1)}$

$|y|=C_2e^{-\frac{1}{2}x^2}e^{-x}$

$y=\pm C_2e^{-\frac{1}{2}x^2}e^{-x}$

$y(-2)=1$

$|1|=C_3e^{-\frac{1}{2}(-2)^2}e^{-(-2)}$

$1=C_3e^{-2}e^{2}$

$1=C_3$

$y=\cancel{\pm}e^{-\frac{1}{2}x^2}e^{-x}$

$y=C_3e^{-\frac{1}{2}x^2}e^{-x}$

general solution

$\boxed{y=e^{-\frac{1}{2}x^2}e^{-x}}$

particular solution

55. $yy'-e^x=0 \quad ; \quad y(0)=4$

$y\cdot\frac{dy}{dx}=e^x$

$\int y\,dy=\int e^x\,dx$

$\frac{1}{2}y^2=e^x+C$

$\frac{1}{2}(4)^2=e^0+C$

$8=1+C$

$C=7$

$\frac{1}{2}y^2=e^x+7$

$\boxed{y^2=2e^x+14}$

6.5

work done by an expanding gas

initial volume: $1\ ft^3$

initial pressure: 500 pounds per ft^2

gas expands to a volume of $2ft^3$

Find the work done by the gas.

Assume pressure is inversely proportional to volume.

$P = \frac{K}{V}$ since $500 = \frac{K}{1}$, $K = 500$

$$W = \int_{V_0}^{V_1} \frac{K}{V}\,dV = \int_1^2 \frac{500}{V}\,dV = 500\ln|v|\Big|_1^2$$

$$= 500\ln 2 \approx \boxed{346.6 \text{ foot-pounds}}$$

Compressing a spring

A force of 750 lb compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring additional 3in.

Hooke's Law: $F(x) = kx \Rightarrow F(x) = 250x$

$750 = k \cdot 3$

$250 = k$

$$W = \int_3^6 250x\,dx = 125x^2\Big|_3^6 = 125(36) - 125(9)$$

$$125(27) = \boxed{3375 \text{ inch-pounds}}$$