2. $\int \tan^3 x \sec x \, dx$

$= \int \tan^2 x \tan x \sec x \, dx$

$= \int (\sec^2 x - 1) \tan x \sec x \, dx$

$u = \sec x$

$du = \sec x \tan x$

$= \int (u^2 - 1) \, du$

$= \frac{u^3}{3} - u + C$

$= \frac{\sec^3 x}{3} - \sec x + C$

3. $\int \frac{\sqrt{16 - x^2}}{x} dx = \int \frac{\sqrt{16 - 16\sin^2\theta}}{\cancel{4}\sin\theta} \cancel{4}\cos\theta \, d\theta$

$x = 4\sin\theta$ $\sin\theta = \frac{x}{4}$

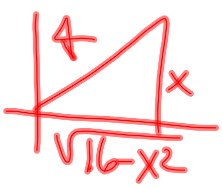
$dx = 4\cos\theta \, d\theta$

$= \int \frac{4\sqrt{1 - \sin^2\theta}}{\sin\theta} \cos\theta \, d\theta$

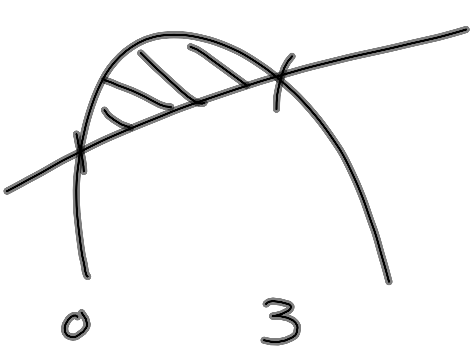
$= \int \frac{4\cos^2\theta}{\sin\theta} d\theta = \int \frac{4(1 - \sin^2\theta)}{\sin\theta} d\theta$

$= \int (4\csc\theta - 4\sin\theta) \, d\theta$

$= -4\ln|\csc\theta + \cot\theta| + 4\cos\theta + C$



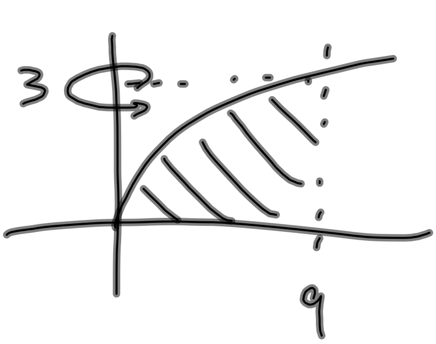
$= -4\ln\left|\frac{4}{x} + \frac{\sqrt{16 - x^2}}{x}\right| + 4 \cdot \frac{\sqrt{16 - x^2}}{4} + C$



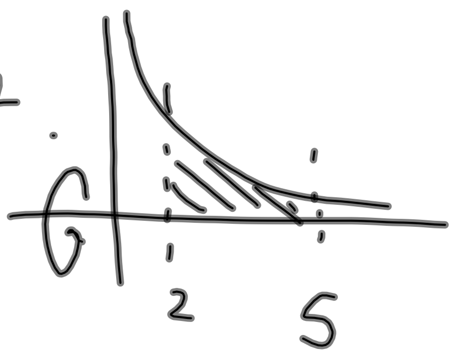
$$x^2-3x=0$$
$$x(x-3)=0$$
$$x=0,3$$

$$\int_0^3 \left[\left(-x^2+4x+5\right)-(x+5)\right]dx = \frac{9}{2}$$

6. $\pi(9)^2(3) - \int_0^3 \pi\left(y^2\right)^2 dy$

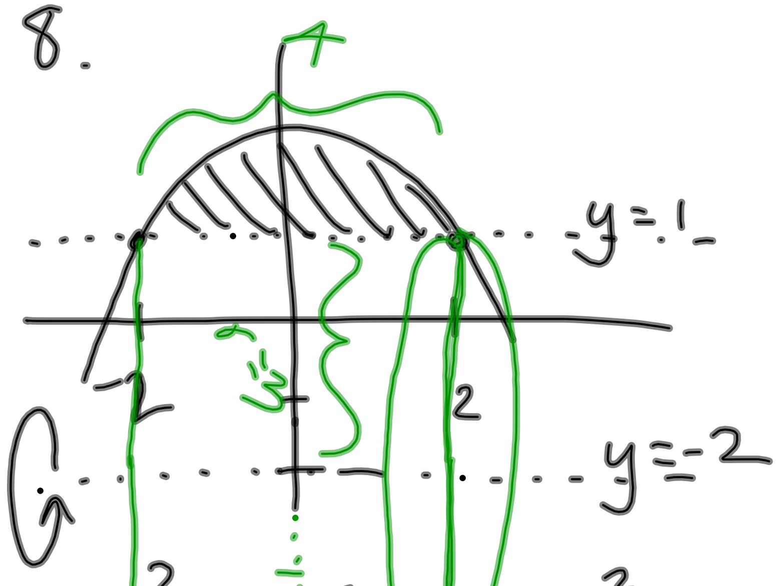


7.



$$\int_2^5 \pi\left(\frac{1}{x}\right)^2 dx$$

8.



$$\int_{-2}^{2} \pi\left(-x^2+5+2\right)^2 dx - \pi\, 3^2 \cdot 4$$

9. $\int_1^8 \sqrt{1+(x^{-1/3})^2}\,dx$

$y = \frac{3}{2}x^{2/3}$

$y' = x^{-1/3}$

10. $\int_4^9 2\pi(2\sqrt{x})\sqrt{1+(x^{-1/2})^2}\,dx$

$y = 2\sqrt{x}$

$y' = x^{-1/2}$

$\frac{1}{\sqrt{x}}$

$= \int_4^9 4\pi\sqrt{x+1}\,dx$

Differential of y : $dy = f'(x)\,dx$

Approximate function value @ $c+\Delta x$

$$f(c+\Delta x) \approx f(c) + f'(c)\Delta x$$

$$\sum_{i=1}^{n} c = cn \ ; \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \ ;$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \ ; \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\text{area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \ ; \quad x_{i-1} \le c_i \le x_i \ ; \quad \Delta x = \frac{b-a}{n}$$

$$\int_a^a f(x)\,dx = 0 \qquad \int k f(x)\,dx = k\int f(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \qquad \int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Power Rule: $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \ n \neq -1$

Mean Value Theorem for Integrals

If f is cts on $[a,b]$, $\exists\, c \in [a,b]$

s.t. $\int_a^b f(x)dx = f(c)(b-a)$

→ Average Value on an interval:

$$\frac{1}{b-a}\int_a^b f(x)dx$$

2nd FTC:

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)dt\right] = f'(g(x)) \cdot g'(x)$$

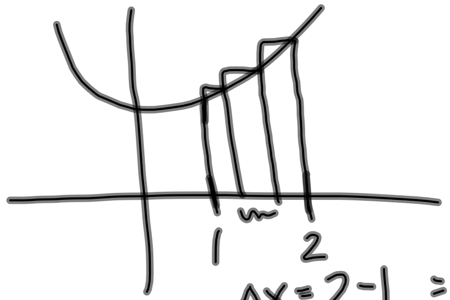
$\int \frac{dx}{x} = \ln|x| + C$; $\int e^x dx = e^x + C$

Test 1

#8 $\int_1^2 (x^2+5)\,dx$ use limit def & check using 1st FTC

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{i}{n}\right)^2 + 5\right] \cdot \frac{1}{n}$$

$$= \lim_{n\to\infty} \ldots$$



$\Delta x = \frac{2-1}{n} = \frac{1}{n}$

What is x-coord of right endpoint of i^{th} rectangle?

$1 + \frac{i}{n}$

- partial fractions
- simple diff. eq. w/ separation of variables & initial conditions

Test 1 #5

$s(0) = 10$ ft

$v(0) = 60$ ft/s

$a(t) = -32$ ft/s^2

$v(t) = \int a(t)\,dt = \int -32\,dt$

$= \underbrace{-32t + 60}$

set $= 0$ & solve for t

max height? occurs when $v(t) = 0$.

$s(t) = \int v(t)\,dt = \int (-32t + 60)\,dt$

$= -16t^2 + 60t + 10$

plug into position