Why does the differential give us a good approximation for the actual change in y?

slope of secant line through $(c, f(c))$ & $(c+\Delta x, f(c+\Delta x))$

$$\frac{f(c+\Delta x)-f(c)}{c+\Delta x - c} = \frac{f(c+\Delta x)-f(c)}{\Delta x}$$

slope of tangent line through $(c, f(c))$ is $f'(c)$

$$\frac{f(c+\Delta x)-f(c)}{\Delta x} \approx f'(c) \quad \text{for small } \Delta x$$

$$f(c+\Delta x) - f(c) \approx f'(c)\Delta x$$

$$\boxed{f(c+\Delta x) \approx f(c) + f'(c)\Delta x}$$

$$T(x) = f(c) + f'(c)\Delta x$$

3.9 #50

Use differentials to approximate $\tan(0.05)$.

Calculator says: $\tan 0.05 = 0.05004\ldots$

$f(c + \Delta x) \approx f(c) + f'(x)dx$

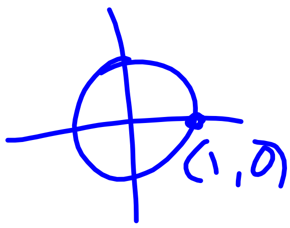
$f(x) = \tan x$; $c = 0$; $\Delta x = dx = 0.05$

$$\tan(0.05) = \tan(0) + [\tan x]' \cdot (0.05) \Big|_{c=0}$$

$$= 0 + \sec^2(0) \cdot 0.05$$

$$= (\sec 0)^2 \cdot 0.05$$

$$= 1^2 \cdot 0.05$$



$$\tan 0.05 \approx 0.05$$

4.1 Antiderivatives

$F(x) = 5x^4$

$f(x) = x^5$ $\qquad [f(x)]' = 5x^4$

particular solution

General solution: $x^5 + C$

Find a general solution and a particular solution to the differential equation.

56. $g'(x) = 6x^2$, $g(0) = -1$

$y' = 6x^2$

$\boxed{y = 2x^3 + C}$ ← general solution

$-1 = 2(0)^3 + C$

$-1 = C$

particular solution: $y = 2x^3 - 1$

$$y = F(x)$$

$$\frac{dy}{dx} = f(x)$$

antiderivative = indefinite integral

$$\int dy = \int f(x)\,dx$$

$$y = \int f(x)\,dx = F(x) + C$$

18. $\int (4x^3 + 6x^2 - 1)\,dx$

$$= \boxed{x^4 + 2x^3 - x + C}$$

24. $\int (\sqrt[4]{x^3} + 1)\,dx$

$$= \int (x^{3/4} + 1)\,dx$$

$$= \boxed{\frac{4}{7}x^{7/4} + x + C}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$x^{-1} = \frac{1}{x}$$

28. $$\int \frac{x^2+2x-3}{x^4}dx = \int\left(\frac{x^2}{x^4}+\frac{2x}{x^4}-\frac{3}{x^4}\right)dx$$

$$= \int\left(x^{-2}+2x^{-3}-3x^{-4}\right)dx$$

$$= \boxed{-x^{-1}-x^{-2}+x^{-3}+C}$$

38. $$\int\left(\theta^2+\sec^2\theta\right)d\theta$$

$$= \boxed{\frac{1}{3}\theta^3+\tan\theta+C}$$

42. $$\int \frac{\cos x}{1-\cos^2 x}dx = \int \frac{\cos x}{\sin^2 x}dx$$

$$= \int \frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}dx = \int \csc x \cot x\, dx$$

$$= \boxed{-\csc x + C}$$

40. $\int \sec y(\tan y - \sec y)\,dy$

$= \int [\sec y \tan y - \sec^2 y]\,dy$

$= \boxed{\sec y - \tan y + C}$

58. $f'(s) = 6s - 8s^3$, $f(2) = 3$

general: $f(s) = 3s^2 - 2s^4 + C$

particular solution: $3 = 3(2)^2 - 2(2)^4 + C$

$\boxed{f(s) = 3s^2 - 2s^4 + 23}$

62. $f''(x) = \sin x$, $f'(0) = 1$, $f(0) = 6$

$f'(x) = -\cos x + c_1$

$$\boxed{f(x) = -\sin x + c_1 x + c_2 \quad \text{general solution}}$$

$1 = -\cos(0) + c_1$; $6 = -\sin(0) + 2 \cdot 0 + c_2$

$1 = -1 + c_1$ $\quad$ $6 = c_2$

$2 = c_1$

particular solution: $\boxed{f(x) = -\sin x + 2x + 6}$

$s(t) =$ position $\quad s \quad m$

$v(t) = s'(t) =$ velocity $\quad \frac{\Delta s}{\Delta t} \quad m/s$

$a(t) = v'(t) = s''(t) =$ acceleration

$\frac{\Delta s}{\Delta t} / \Delta t \quad m/s^2$

72. 1600 m $a = -9.8\ m/s^2$

$a(t) = v'(t) = s''(t)$

$s''(t) = -9.8$

$s'(t) = -9.8t + V_0$

$s(t) = \frac{-9.8t^2}{2} + V_0 t + S_0$

$0 = \frac{-9.8}{2}t^2 + 1600$

$\sqrt{\frac{3200}{9.8}} = t \approx \boxed{18.1s}$

80. $a(t) = \cos t$, $t > 0$

@ $t = 0$; position is $x = 3$ $s(0) = 3$

$v(0) = 0$

a) find velocity & position functions

b) find value(s) of t for which the particle is at rest. $t = 0, \ldots, \pi k, k \in \mathbb{Z}$

$\mathbb{Z}$ is the set of Integers

$v(t) = \sin t + C_1 \Rightarrow v(t) = \sin t$

$0 = \sin(0) + C_1$

$s(t) = -\cos t + C_1 t + C_2 \Rightarrow s(t) = -\cos t + 4$

$3 = -\cos(0) + C_2$

$4 = C_2$

Homework:

Already assigned:

3.9 #5, 9; 11-19 odd; 45, 49

New:

4.1 #5-33 odd; 55-61 odd; 67, 83