3.9 #5, 9; 11-19 odd; 45, 49
4.1 #5-33 odd; 55-61 odd; 67, 83

Homework questions?

(<-- note to self, check for homework completion & assign 1st hw grade!)

$$y' = f(x)$$

$$y = \int f(x) \cdot dx + C$$

$$y' = f'(x)$$

$$dy = f'(x) \cdot dx$$

4.2 Area

Sigma Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

Summation Formulas

1. $\sum_{i=1}^{n} c = nc$

2. $\sum_{i=1}^{n} i = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\ldots+n^3 = \frac{n^2(n+1)^2}{4}$

Use sigma notation to write the sum.

8. $\frac{5}{1+1} + \frac{5}{1+2} + \frac{5}{1+3} + \ldots + \frac{5}{1+15}$

$$\sum_{i=1}^{15} \frac{5}{1+i}$$

14. $\left(\frac{1}{n}\right)\sqrt{1-\left(\frac{0}{n}\right)^2} + \ldots + \left(\frac{1}{n}\right)\sqrt{1-\left(\frac{n-1}{n}\right)^2}$

$$\sum_{i=1}^{n} \frac{1}{n}\sqrt{1-\left(\frac{i-1}{n}\right)^2} = \sum_{i=1}^{n} \frac{1}{n}\sqrt{1-\left(\frac{n-i}{n}\right)^2}$$

Use summation properties to evaluate the sum.

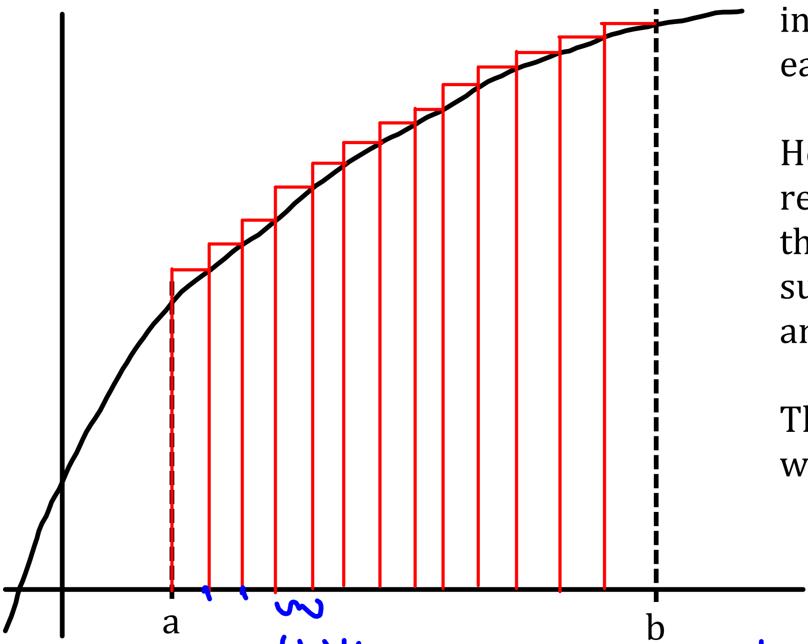
$$20.\ \sum_{i=1}^{10} i(i^2+1)$$

$$= \sum_{i=1}^{10}(i^3+i) = \sum_{i=1}^{10} i^3 + \sum_{i=1}^{10} i =$$

$$= \frac{10^2(10+1)^2}{4} + \frac{10(10+1)}{2}$$

$$= \frac{12100}{4} + \frac{110}{2} \quad \frac{6050+110}{2} = \frac{6160}{2} = \boxed{3080}$$

4.2 Area



width

$\frac{b-a}{n}$

$a + \frac{b-a}{n}$

$a + 2\cdot\frac{b-a}{n}$

Divide the interval [a,b] into n equal subintervals, each of width (b-a)/n.

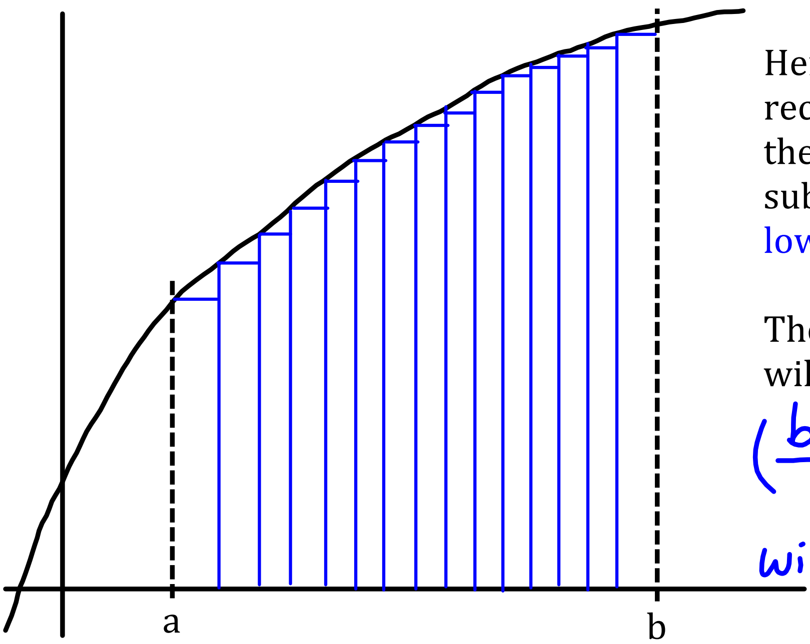
Here, the height of a rectangle is determined by the right endpoint of each subinterval; this is called an upper sum.

The area of each rectangle will be:

width · height of i^{th} rectangle

$$\frac{b-a}{n} \cdot f\left(a + i\cdot\frac{b-a}{n}\right)$$

↑ function value @ rt. endpt.



Here, the height of a rectangle is determined by the left endpoint of each subinterval; this is called a lower sum.

The area of each rectangle will be:

$$\left(\frac{b-a}{n}\right)\cdot f\left(a+(i-1)\frac{b-a}{n}\right)$$

width × f(right endpoint of i^{th} rect)

$$\left(\frac{b-a}{n}\right)\cdot f\left(a+(i-1)\cdot\frac{b-a}{n}\right)$$

Lower sum: $s(n)=\sum_{i=1}^{n} f(m_i)\,\Delta x$

Upper sum: $S(n)=\sum_{i=1}^{n} f(M_i)\,\Delta x$

$f(m_i)$ = minimum function value in an interval

$f(M_i)$ = Maximum function value in an interval

$\Delta x = \frac{b-a}{n}$

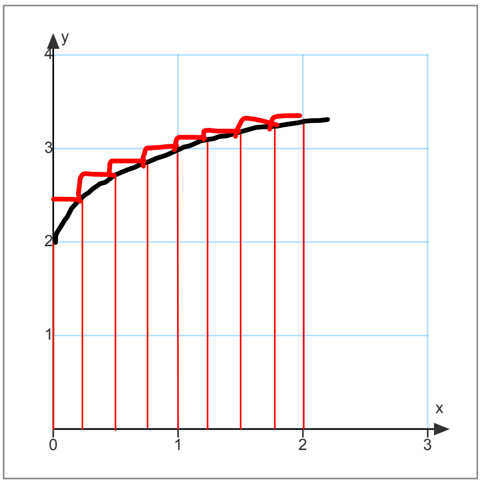
$s(n) \leq S(n)$

Area of the region bounded by the graph of f, the x-axis, & the lines x=a & x=b is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \,,\quad x_{i-1} \le c_i \le x_i$$

where $\Delta x = \frac{b-a}{n}$.

28. $y = \sqrt{x} + 2$



use upper sum & lower sum to approximate area under the curve.

$\Delta x = \frac{2-0}{8} = \frac{1}{4}$

upper sum = 6.16

$\frac{1}{4}\left(\sqrt{\tfrac{1}{4}}+2+\sqrt{\tfrac{1}{2}}+2+\sqrt{\tfrac{3}{4}}+2+\sqrt{1}+2+\sqrt{\tfrac{5}{4}}+2+\sqrt{\tfrac{6}{4}}+2+\sqrt{\tfrac{7}{4}}+2+\sqrt{2}+2\right)$

lower sum = 5.68

$\frac{1}{4}\left(\sqrt{0}+2+\sqrt{\tfrac{1}{4}}+2+\ldots+\sqrt{\tfrac{7}{4}}+2\right)$

$$\lim_{n\to\infty} S(n)$$

32. $S(n) = \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{64(\cancel{2}\cancel{n^3})}{3\cancel{6}\cancel{n^3}} = \boxed{\frac{64}{3}}$$

rewrite without summation notation

36. $\sum_{j=1}^{n} \frac{4j+3}{n^2} = \frac{1}{n^2}\sum_{j=1}^{n}(4j+3)$

$$= \frac{1}{n^2}\left[4\sum_{j=1}^{n} j + \sum_{j=1}^{n} 3\right] =$$

$$= \frac{1}{n^2}\left[\overset{2}{\cancel{4}}\cdot\frac{n(n+1)}{\cancel{2}} + 3n\right] = \frac{2n^2+2n+3n}{n^2}$$

$$= \frac{2n^2+5n}{n^2}$$

$$= \frac{\cancel{n}(2n+5)}{\cancel{n}\cdot n}$$

$$= \boxed{\frac{2n+5}{n}}$$

42. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{2i}{n}\right)^2 \left(\frac{2}{n}\right)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right)\left(\frac{2}{n}\right)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}+\frac{8i}{n^2}+\frac{8i^2}{n^3}\right) =$

$= \lim_{n\to\infty} \left[\frac{2}{n}\sum_{i=1}^{n} 1 + \frac{8}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$

$= \lim_{n\to\infty} \left[\frac{2}{n}\cdot n + \frac{8}{n^2}\cdot\frac{n(n+1)}{2} + \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$

$= 2 + 4 + \frac{8}{3}$

$= \boxed{\frac{26}{3}}$

4.2 Homework:

#7-19odd; 27-37odd; 41,43,47,53