44. $$\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

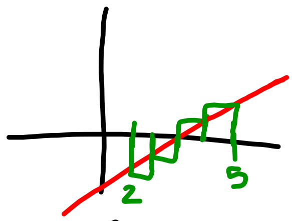
$$=\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(1+\frac{6i}{n}+\frac{12i^2}{n^2}+\frac{8i^3}{n^3}\right)$$

$$=\lim_{n\to\infty}\left[\left(\frac{2}{n}\sum_{i=1}^{n}1\right)+\left(\frac{12}{n^2}\sum_{i=1}^{n}i\right)+\left(\frac{24}{n^3}\sum_{i=1}^{n}i^2\right)+\left(\frac{16}{n^4}\sum_{i=1}^{n}i^3\right)\right]$$

$$=\lim_{n\to\infty}\left[\frac{2}{n}\cdot\frac{n}{1}+\frac{12}{n^2}\cdot\frac{n(n+1)}{2}+\frac{24}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{16}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right]$$

$$=2+6+8+4=\boxed{20}$$

48. $y=3x-4$, $[2,5]$



$$\text{Area}=\lim_{n\to\infty}\sum_{i=1}^{n}f(c_i)\Delta x \quad , \quad x_i \le c_i \le x_{i+1},$$

$$\Delta x=\frac{b-a}{n}=\frac{5-2}{n}=\frac{3}{n}$$

$2,\ 2+\frac{3}{n},\ 2+\frac{6}{n},\ 2+\frac{9}{n}$

left endpoint $2+\frac{i-1}{1}\cdot\frac{3}{n}$

right endpoint $2+\frac{3i}{n}$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left[3\left(2+\frac{3i}{n}\right)-4\right]\cdot\frac{3}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{18}{n}+\frac{27i}{n^2}-\frac{12}{n}\right)=$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{6}{n}+\frac{27i}{n^2}\right)=$$

$$=\lim_{n\to\infty}\left[\frac{6}{n}\sum_{i=1}^{n}1+\frac{27}{n^2}\sum_{i=1}^{n}i\right]=$$

$$=\lim_{n\to\infty}\left[\frac{6}{n}\cdot n+\frac{27}{n^2}\cdot\frac{n(n+1)}{2}\right]=$$

$$=6+\frac{27}{2}=\boxed{19.5}$$

56. $y = x^2 - x^3 \quad [-1, 0]$

$$\boxed{\Delta x = \frac{b-a}{n}} = \frac{0-(-1)}{n} = \frac{1}{n}$$

$$\boxed{c_i = a + i\Delta x} = -1 + \frac{i}{n}$$

$$\text{area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\cdot\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-1+\frac{i}{n}\right)^2 - \left(-1+\frac{i}{n}\right)^3\right]\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 - \frac{2i}{n} + \frac{i^2}{n^2} - \left(-1 + \frac{3i}{n} - \frac{3i^2}{n^2} + \frac{i^3}{n^3}\right)\right]\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[2 - \frac{5i}{n} + \frac{4i^2}{n^2} - \frac{i^3}{n^3}\right]\cdot\frac{1}{n} =$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\sum_{i=1}^{n} 1 - \frac{5}{n^2}\sum_{i=1}^{n} i + \frac{4}{n^3}\sum_{i=1}^{n} i^2 - \frac{1}{n^4}\sum_{i=1}^{n} i^3\right] =$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\cdot n - \frac{5}{n^2}\cdot\frac{n(n+1)}{2} + \frac{4}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{1}{n^4}\cdot\frac{n^2(n+1)^2}{4}\right] =$$

$$= 2 - \frac{5}{2} + \frac{4}{3} - \frac{1}{4} = \frac{24}{12} - \frac{30}{12} + \frac{16}{12} - \frac{3}{12} = \boxed{\frac{7}{12}}$$

4.3 Riemann Sums & Definite Integrals

$$\sum_{i=1}^{n} f(c_i)\Delta x_i \quad , \quad x_{i-1} \le c_i \le x_i ,$$

where c_i is any point in the i^{th} subinterval ; $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ is called the Riemann Sum of f.

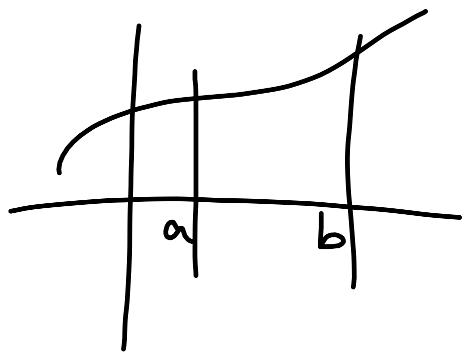
$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)\,dx$$

called the <u>definite integral</u> of f from a to b.

<u>Properties</u>

If $f(a)$ is defined,

$$\int_a^a f(x)\,dx = 0$$

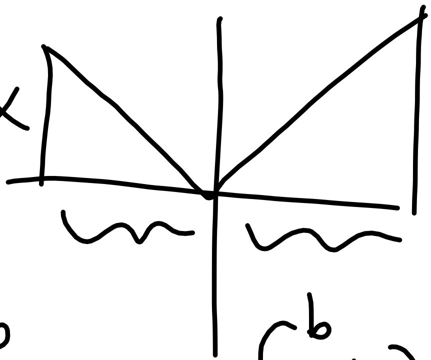


If f is integrable on $[a,b]$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$$



$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

If $f(x) \geq 0$,

$$\int_a^b f(x)\,dx \geq 0$$

If $f(x) \leq g(x)$

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

typo on p. 272

8. $\int_{-1}^{2} (3x^2+2)\,dx$

$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\Delta x)\cdot\Delta x$

$\Delta x = \frac{2-(-1)}{n} = \frac{3}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(3\left(-1+\frac{3i}{n}\right)^2+2\right)\cdot\frac{3}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{9}{n}\left(1-\frac{6i}{n}+\frac{9i^2}{n^2}\right)+\frac{6}{n}\right)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{15}{n}\right.$ . . . .

Homework for Test #1

HW#1 (submitted Mon. 11/11)
- 3.9 #5, 9; 11-19 odd; 45, 49
- 4.1 #5-33 odd; 55-61 odd; 67, 83

HW#2 (due Mon. 11/18?)
- 4.2 #7-19odd; 27-37odd; 41,43,47,53
- 4.3 #7,17,37,43,45

Quiz #1 Wednesday, 11/13

Test #1 Tuesday, 11/19