Find the differential dy. $dy = f'(x)dx$

1. $y = 5x^3 + 7$

$dy = 15x^2 dx$

2. $y = 4x - \tan x$

$dy = (4 - \sec^2 x)dx$

Find the antiderivative.

3. $\int (x^4 - 3)dx$

$\frac{x^5}{5} - 3x + C$

4. $\int (\csc^2\theta - \sin\theta)\, d\theta$

$-\cot\theta + \cos\theta + C$

Approximate the function value using differentials.

5. $\sqrt{104}$

Hint: $f(c + \Delta x) \approx f(c) + f'(c)\Delta x$

$f(x) = \sqrt{x}$; $c = 100$; $\Delta x = 4$

$= x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\sqrt{100+4} \approx \sqrt{100} + \frac{1}{2\sqrt{100}} \cdot 4$

$= 10 + \frac{2}{10} = \boxed{10.2}$

$= \frac{51}{5}$

4.3

$\int_a^c = \int_a^b + \int_b^c$ $\qquad \int_a^b = -\int_b^a$

44. Given $\int_{-1}^{1} f(x)dx = 0$ & $\int_0^1 f(x)dx = 5$

$\int_{-1}^{1} = \int_{-1}^{0} + \int_0^1$

(a) $\int_{-1}^{0} f(x)dx = \int_{-1}^{1} f(x)dx - \int_0^1 f(x)dx = 0 - 5 = \boxed{-5}$

(b) $\int_0^1 f(x)dx - \int_{-1}^{0} f(x)dx = 5 - (-5) = \boxed{10}$

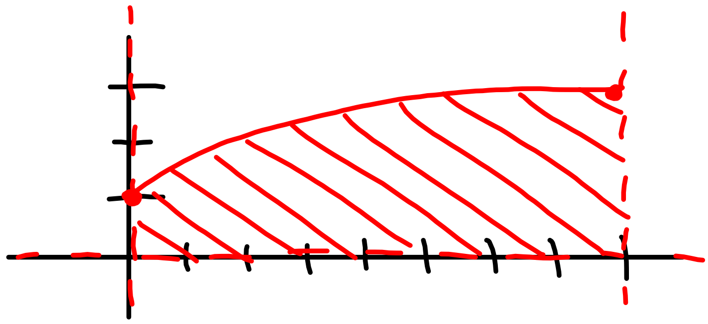
(c) $\int_{-1}^{1} 3f(x)dx = 3\int_{-1}^{1} f(x)dx = 3 \cdot 0 = \boxed{0}$

(d) $\int_0^1 3f(x)dx = 3 \cdot 5 = \boxed{15}$

(e) $\int_1^0 f(x)dx = -\int_0^1 f(x)dx = -5$

4.4 find area of region bounded by ...

42. $y = 1 + \sqrt[3]{x}$, $x = 0$, $x = 8$, $y = 0$



$$\int_0^8 (1 + x^{1/3})\, dx$$

$$= x + \frac{3}{4} x^{4/3} \Big|_{x=0}^{8}$$

$$= 8 + \frac{3}{4} \cdot (\sqrt[3]{8})^4 - 0 = \boxed{20}$$

4.3 calculate using limit def.

6. $$\int_1^3 3x^2 dx = \lim_{n\to\infty} \sum_{i=1}^{n} 3\left(1 + \frac{2i}{n}\right)^2 \cdot \frac{2}{n}$$

$$f\left(a + \frac{(b-a)}{n} i\right) \cdot \frac{b-a}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 3\left(1 + \frac{4i}{n} + \frac{4i^2}{n^2}\right) \cdot \frac{2}{n} =$$

$$= \lim_{n\to\infty} \left[\frac{6}{n} \sum_{i=1}^{n} 1 + \frac{24}{n^2} \sum_{i=1}^{n} i + \frac{24}{n^3} \sum_{i=1}^{n} i^2\right] =$$

$$= \lim_{n\to\infty} \left[\frac{6}{n} \cdot n + \frac{24}{n^2} \cdot \frac{n(n+1)}{2} + \frac{24}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right] =$$

$$= 6 + 12 + 8 = \boxed{26}$$

4.2

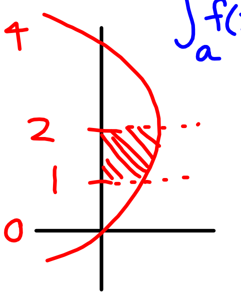
40. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{4i}{n^2} = \lim_{n\to\infty}\frac{4}{n^2}\sum_{i=1}^{n} i =$

$= \lim_{n\to\infty}\frac{4}{n^2}\cdot\frac{n(n+1)}{2} = \boxed{2}$

60. $f(y) = 4y - y^2 \;,\; 1 \le y \le 2$

$= y(4-y)$



$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right)\cdot\frac{b-a}{n}$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[4\left(1+\frac{i}{n}\right) - \left(1+\frac{i}{n}\right)^2\right]\cdot\frac{1}{n} =$

$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{4}{n} + \frac{4i}{n^2} - \frac{1}{n} - \frac{2i}{n^2} - \frac{i^2}{n^3}\right] =$

$= \lim_{n\to\infty}\left[\frac{3}{n}\sum_{i=1}^{n} 1 + \frac{2}{n^2}\sum_{i=1}^{n} i - \frac{1}{n^3}\sum_{i=1}^{n} i^2\right] =$

$= \lim_{n\to\infty}\left[\frac{3}{n}\cdot n + \frac{2}{n^2}\cdot\frac{n(n+1)}{2} - \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right] =$

$= 3 + 1 - \frac{1}{3} = \boxed{\frac{11}{3}}$

$\int_1^2 (4y - y^2)dy = 2y^2 - \frac{1}{3}y^3\Big|_1^2 = 8 - \frac{8}{3} - \left(2 - \frac{1}{3}\right) = 6 - \frac{7}{3} = \frac{11}{3}$ ✓

Homework for **Test #1 (Tuesday, 11/19)**

HW#1 (submitted Mon. 11/11)
- 3.9 #5, 9; 11-19 odd; 45, 49
- 4.1 #5-33 odd; 55-61 odd; 67, 83

HW#2 (due Mon. 11/18)
- 4.2 #7-19odd; 27-37odd; 41,43,47,53
- 4.3 #7,17,37,43,45
- 4.4 #13, 15, 23, 31
- **Test #1 Practice Problems**