Homework for **Test #1 (Tuesday, 11/19)**

HW#1 (submitted Mon. 11/11)
- 3.9 #5, 9; 11-19 odd; 45, 49
- 4.1 #5-33 odd; 55-61 odd; 67, 83

HW#2 (due ~~Mon.~~ Tues 11/~~18~~ 19)
- **4.2 #7-19odd; 27-37odd; 41,43,47,53**
- **4.3 #7,17,37,43,45**
- **4.4 #13, 15, 23, 31**
- **Test #1 Practice Problems**

Find the differential dy.

1. $y = x(1 - \cos x)$
2. $y = \sqrt{36 - x^2}$

$dy = f'(x)dx$

3. Use differentials to approximate the value of the expression.
$\sqrt[3]{63}$ $f(c+\Delta x) \approx f(c) + f'(c) \cdot \Delta x$

Find the indefinite integral and check the result by differentiation.

4. $\int \frac{x^2-3x+4}{x^4} dx = \int (x^{-2} - 3x^{-3} + 4x^{-4}) dx$
5. $\int \left(\frac{4}{x^4} + \sin x\right) dx$
6. $\int (5\cos x - 2\sec^2 x)\, dx$

7. Solve the differential equation.
   $f'(x) = 6x^2 - \cos x$, $f(0) = 1$

8. Find the particular solution of the differential equation $f'(x) = -2x$ whose graph passes through the point $(-1,1)$.

9. Find the particular solution of the differential equation $f''(x) = 6(x-1)$ whose graph passes through the point $(2,1)$ and is tangent to the line $3x - y - 5 = 0$ at that point.

10. A ball is thrown vertically from a height of 5 feet with initial velocity of 50 feet per second. How high will the ball go? (Use $a(t) = -32$ feet per second per second as the acceleration due to gravity. Neglect air resistance.)

11. The rate of growth dP/dt of a population of bacteria is proportional to the cube root of t, where P is the population size and t is the time in days $(0 \le t \le 10)$. That is, $\frac{dP}{dt} = k\sqrt[3]{t}$. The initial size of the population is 1000. After 1 day the population has grown to 1100. Estimate the population after 10 days.

12. A particle, initially at rest, moves along the x-axis such that its acceleration at time $t > 0$ is given by $a(t) = \sin t$. At time $t = 0$, its position is $x = 5$.
    (a) Find the velocity and position function for the particle.
    (b) Find the values of t for which the particle is at rest.

$-32t + 50 = 0$

$t = \frac{50}{32} = \frac{25}{16}$

10. $s''(t) = -32$

$s'(t) = -32t + 50$

$s(t) = -16t^2 + 50t + 5$

$s\left(\frac{25}{16}\right) = -16\left(\frac{25}{16}\right)^2 + 50\left(\frac{25}{16}\right) + 5 = \boxed{}$

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(a) Find the velocity and position function for the particle.
(b) Find the values of t for which the particle is at rest.

11. $\frac{dP}{dt} = k \cdot t^{1/3}$

$P = \frac{3k}{4} t^{4/3} + 1000$

$1100 = \frac{3}{4}k + 1000$

$\frac{400}{3} = k$

$P(t) = 100t^{4/3} + 1000$

$P(10) = 100\sqrt[3]{10^4} + 1000$

$= 1000\sqrt[3]{10} + 1000$

$\approx$

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(a) Find the velocity and position function for the particle.
(b) Find the values of t for which the particle is at rest.

12. $a(t) = s''(t) = \sin t$

$v(t) = s'(t) = -\cos t + C_1$

$0 = -\cos(0) + C_1$

$1 = C_1$

$\boxed{v(t) = -\cos t + 1}$

$s(t) = -\sin t + t + C_2$

$5 = -\sin(0) + 0 + C_2$

$5 = C_2$

$\boxed{s(t) = -\sin t + t + 5}$

$0 = -\cos t + 1$

$\cos t = 1$

$t = 2\pi k, \ k \in \mathbb{Z}_+$

Evaluate the sum.

13. $\sum_{i=1}^{20}(3i^2 + 2i + 4)$
14. $\sum_{i=1}^{40} i(i^2 + 1)$

$\sum 1 = n$

$\sum i = \frac{n(n+1)}{2}$

$\sum i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum i^3 = \frac{n^2(n+1)^2}{4}$

Find the limit.

15. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-2)^2$

16. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3i}{n^2}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i^2 - 4i + 4)$

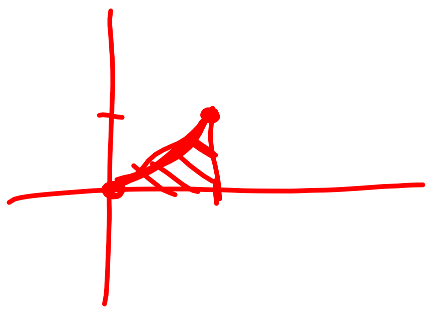
$= \lim_{n\to\infty} \left[\frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{4}{n^3} \cdot \frac{n(n+1)}{2} + \frac{4}{n^3} \cdot n \right]$

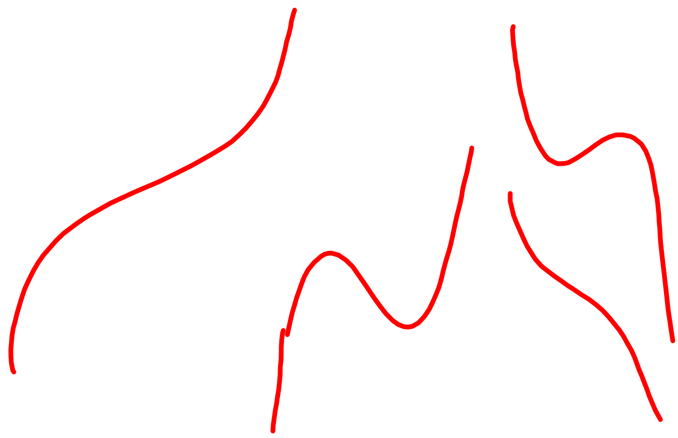
$= \boxed{\frac{1}{3}}$

Use the limit process to find the area of the region between the graph of the function and the x-axis over the indicated interval. Sketch the region.

17. $y = 2x - x^3$, $[0,1]$
18. $y = x^2 - x^3$, $[-1,0]$

17. $\lim\limits_{n\to\infty} \sum_{i=1}^{n} \left[2\left(0+\frac{1}{n}i\right) - \left(0+\frac{1}{n}i\right)^3 \right] \cdot \frac{1}{n}$





Use the limit process to find the area of the region between the graph of the function and the y-axis over the indicated interval. Sketch the region.

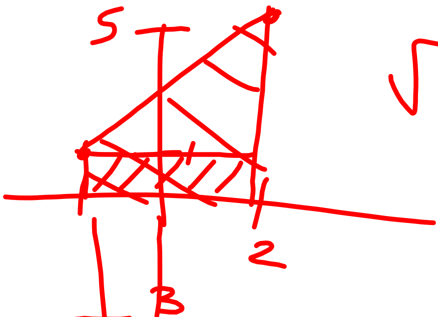
19. $f(y) = y^2$, $0 \le y \le 3$

20. Evaluate the definite integral by the limit definition.

$$\int_1^3 (2x+5)\,dx$$

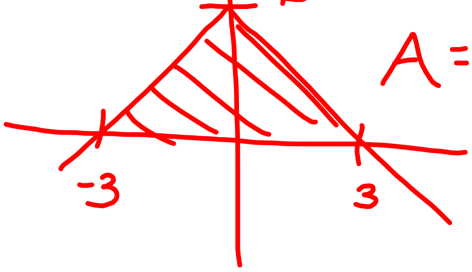
Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

21. $\int_{-2}^{2}(x+3)dx$
22. $\int_{-3}^{3}(3-|x|)dx$
23. $\int_{-2}^{2}\sqrt{4-x^2}dx$



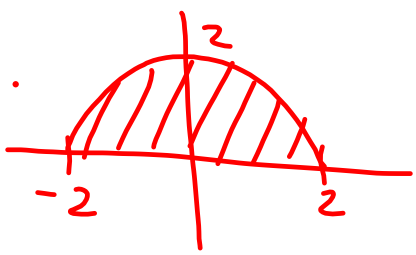
$\sqrt{a^2-x^2}$

22.



$A = \frac{1}{2}(6)\cdot 3 = \boxed{9}$

23.



$A = \frac{1}{2}\pi(2)^2 = \boxed{2\pi}$

Express the limit as a definite integral on the interval [2,3], where c_i is any point in the ith subinterval.

a b

24. $\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} (c_i{}^2 + 5)\,\Delta x_i$
25. $\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} c_i(2c_i{}^2 + 1)\,\Delta x_i$

24. $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(a+\frac{(b-a)}{n}i\right)^2 + 5\right] \cdot \frac{b-a}{n}$

$a + \Delta x i$

right endpoint of n^{th} interval

Δx

$\int_2^3 (x^2+5)\,dx$

25. $\int_2^3 x(2x^2+1)\,dx$

Given $\int_0^3 f(x) = 5$ and $\int_3^7 f(x) = 11$, find

26. $\int_3^0 4f(x)dx$ $= -\int_0^3$ _____
27. $\int_3^3 10f(x)dx$ $= 0$
28. $\int_7^0 f(x)dx$ $= -\int_0^7 = -\left(\int_0^3 + \int_3^7\right)$
29. $\int_0^7 4f(x)dx$ $= \int_0^3 + \int_3^7$
30. $\int_7^3 -f(x)dx$

$= -\int_3^7$ ___

Given $\int_1^5 f(x) = 5$ and $\int_1^5 g(x) = 14$, find

31. $\int_1^5 [4f(x) - g(x)]\, dx$
32. $\int_1^5 [5f(x) + g(x)]\, dx$

1. Use differentials to approximate the value of the expression. Give an exact answer as well as an approximate answer to five decimal places.

$$\cos\frac{19\pi}{40}$$

2. Find the indefinite integral and check the result by differentiation.

$$\int (\sec x \tan x + 10 \sin x) dx$$

3. Evaluate the definite integral. Give a simplified exact answer.

$$\int_1^2 \frac{x^2 - x + 5}{x^4} dx$$

4. Solve the differential equation.

$f''(x) = 3x^2 - \sin x$, $f(0) = 1$, $f'(0) = 2$

5. A ball is thrown vertically from a height of 10 feet with initial velocity of 60 feet per second. How high will the ball go? Use $a(t) = -32\frac{ft}{s^2}$ as the acceleration due to gravity. Give a simplified exact answer.

6. Evaluate the sum.

$$\sum_{i=1}^{40}(i^2 + 2i)$$

7. Use the limit process to find the area of the region between the graph of the function and the x-axis over the indicated interval. Sketch the region. Give a simplified exact answer.

$y = 4x - x^2$, $[0,2]$

$f'(x) = x^3 + \cos x + c_1$

$2 = 0 + 1 + c_1 \Rightarrow c_1 = 1$

$f(x) = \frac{1}{4}x^4 + \sin x + x + c_2$

$1 = c_2$

$f(x) = \frac{1}{4}x^4 + \sin x + x + 1$

General Solution:

$f(x) = \frac{1}{4}x^4 + \sin x + c_1 x + c_2$

Particular solution:

$f(x) = \frac{1}{4}x^4 + \sin x + x + 1$

8. Evaluate the definite integral by the limit definition. Give a simplified exact answer.
Check your answer by computing the definite integral using the Fundamental Theorem of Calculus.

$$\int_1^2 (x^2 + 5)dx$$

9. Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

$a)\ \int_{-2}^{4} (x + 4)dx$

$b)\ \int_{-4}^{4} \sqrt{16 - x^2}$

10. Express the limit as a definite integral on the interval $[1,3]$, where c_i is any point in the ith subinterval. Do not solve.

$a)$ $\lim_{\|x\|\to 0} \sum_{i=1}^{n} (5{c_i}^2 + 2)\Delta x_i$

$b)$ $\lim_{\|x\|\to 0} \sum_{i=1}^{n} {c_i}^3(5{c_i}^2 + 5)\Delta x_i$

A. Find the function whose tangent has slope $5x^4 - x + 5$ for each value of x, and whose graph passes through the point $(0,8)$.

B. Use the limit process to find the area of the region bounded by

$x = 5y - y^2\,,\ x = 0\,,\ y = 2\,,\ y = 5$

$f'(x) = 5x^4 - x + 5$

$f(x) = x^5 - \frac{1}{2}x^2 + 5x + C$

$8 = C$

$\boxed{f(x) = x^5 - \frac{1}{2}x^2 + 5x + 8}$