1. $f(c+\Delta x) \approx f(c) + f'(c)\cdot \Delta x$

$$\sin\frac{39\pi}{20} \approx \sin 2\pi + \cos 2\pi \cdot \frac{-\pi}{20}$$

$f(x) = \sin x$

$f'(x) = \cos x$

$c = \frac{40\pi}{20} = 2\pi$

$\Delta x = -\frac{\pi}{20}$

$$= 0 + 1 \cdot \frac{-\pi}{20}$$

$$= \boxed{-\frac{\pi}{20}}$$

2. $$\int_1^2 \left(x^{-2} - 3x^{-3} + 5x^{-4}\right)dx$$

$$= \left.\frac{-1}{x} + \frac{3}{2x^2} - \frac{5}{3x^3}\right|_1^2$$

$$= \boxed{\frac{5}{6}}$$

3. $-\cot x + 3\cos x + C$

4. general: $f(x) = -\frac{1}{3}x^4 - \cos x + C_1 x + C_2$

particular: $f(x) = -\frac{1}{3}x^4 - \cos x + 2x + 4$

5. $v(t) = s'(t)$

$s(t) = \int s'(t)\,dt$

$\Delta s = \int_1^4 \sqrt{t}\,dt = \frac{2}{3}t^{3/2}\Big|_1^4 = \boxed{\frac{14}{3}}$

6. 208165

$\left(1+\frac{2i}{n}\right)^2 = 1 + \frac{4i}{n} + \frac{4i^2}{n^2}$

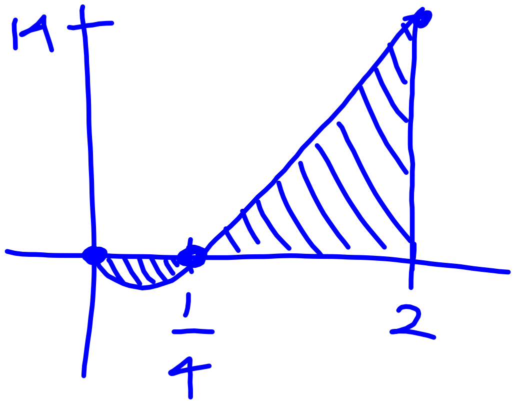
7. $\lim_{n\to\infty}\left[\frac{4}{n}\cdot n - \frac{16}{n^2}\cdot\frac{n(n-1)}{2} - \frac{16}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right.$

$= \boxed{\frac{-28}{3}}$

$\int_1^3(-2x^2+4)\,dx = -\frac{2}{3}x^3+4x\Big|_1^3 \;\ldots$

8. $y = 4x^2 - x \quad , [0,2]$

$x(4x-1)$



$$\int_0^2 f(x)\,dx$$

$$= -\int_0^{1/4} f(x)\,dx + \int_{1/4}^2 f(x)\,dx$$

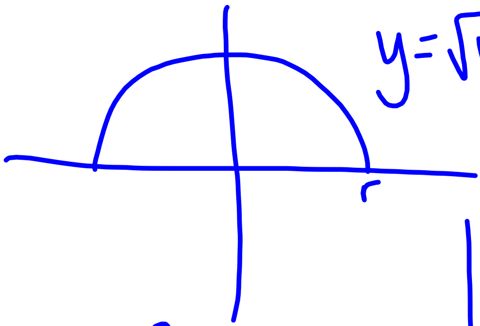
$$= \boxed{\frac{139}{16}}$$

9. $y = \sqrt{81 - x^2}$ $\quad (x-h)^2 + (y-k)^2 = r^2$

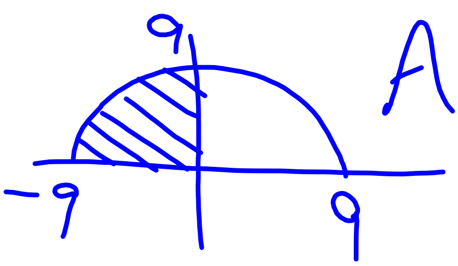
$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$y = \pm\sqrt{r^2 - x^2}$

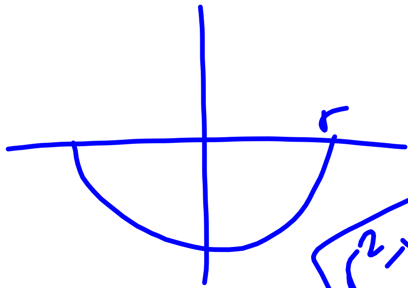


$y = \sqrt{r^2 - x^2}$



$$A = \frac{1}{4}\pi r^2$$

$$= \boxed{\frac{81\pi}{4}}$$



$y = -\sqrt{r^2 - x^2}$

10. $\int_1^3 \left(4x^5 + 2\cos \pi x\right) dx$

A. $\frac{1}{25}$ mi

B. $f(x) = \frac{1}{20}x^5 + \frac{1}{6}x^4 - \frac{22}{3}x + \frac{57}{5}$

4.4 – The Fundamental Theorem of Calculus

<u>1st Fundamental Theorem of Calculus:</u>

If a function f is continuous on the closed interval $[a,b]$ and F is an antiderivative of f on the interval $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

<u>Mean Value Theorem for Integrals:</u>

If a function f is continuous on the closed interval $[a,b]$, then there exists a number c in the closed interval $[a,b]$ such that

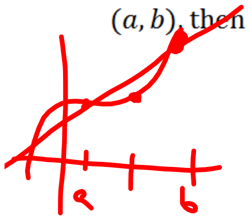
$F'(x) = f(x)$

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Recall the <u>Mean Value Theorem:</u>

If f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), then there exists a number c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



$F(b) - F(a) = F'(c) \cdot (b-a)$

46. $f(x) = \frac{9}{x^3}$, $[1,3]$

MVTI holds since f is cts. on $[1,3]$

$\Rightarrow \exists\, c \in [1,3]$ s.t.

$$\int_1^3 9x^{-3}\,dx = \frac{9}{c^3}(3-1)$$

$$-\frac{9}{2}x^{-2}\Big|_1^3 = \frac{18}{c^3}$$

$$\frac{-9}{2\cdot 3^2} - \left(\frac{-9}{2\cdot 1^2}\right) = \frac{18}{c^3}$$

$$-\frac{1}{2} + \frac{9}{2} = \frac{18}{c^3}$$

$$4c^3 = 18$$

$$c^3 = \frac{9}{2}$$

$$c = \boxed{\sqrt[3]{9/2}} \in [1,3] \checkmark$$

$$1 < \frac{9}{2} < 8$$

$$\sqrt[3]{1} < \sqrt[3]{9/2} < \sqrt[3]{8}$$

$$1 < \sqrt[3]{9/2} < 2$$

Average value of a function on an interval:

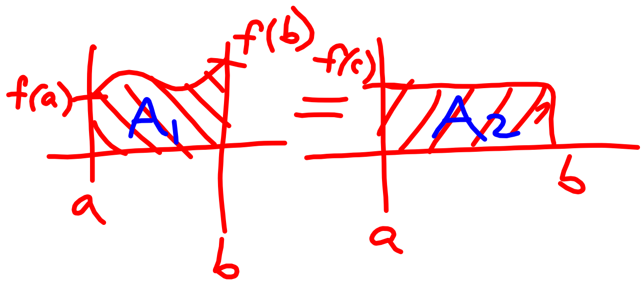
If f is integrable on the closed interval $[a, b]$, then the average value of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$$

MVTI:
$\exists\, c \in [a,b]$ s.t.

$$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$$

Multiply both sides by $\frac{1}{b-a}$:



50. $f(x) = \frac{4(x^2+1)}{x^2}$, $[1,3]$

avg. value: $\frac{1}{3-1}\int_1^3 (4 + 4x^{-2})\,dx$

$= \frac{1}{2}\left[4x - \frac{4}{x}\right]_1^3 =$

$= \frac{1}{2}\left[4(3) - \frac{4}{3} - (4-4)\right] = \frac{1}{2}\left[12 - \frac{4}{3}\right] =$

$= 6 - \frac{2}{3} = \boxed{\frac{16}{3}}$

The Second Fundamental Theorem of Calculus:
If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

"Fix x"

76. $F(x) = \int_0^x t(t^2+1)\,dt = \int_0^x (t^3+t)\,dt$

rewrite $F(x)$ as a function of x.

$\left[\frac{t^4}{4} + \frac{t^2}{2}\right]_0^x = \frac{x^4}{4} + \frac{x^2}{2} - \left(\frac{0^4}{4} + \frac{0^2}{2}\right)$

verify using 2nd FTC.

$\int_0^x t(t^2+1)\,dt = \frac{x^4}{4} + \frac{x^2}{2}$

$\frac{d}{dx}\int_0^x t(t^2+1)\,dt = \frac{d}{dx}\left(\frac{x^4}{4} + \frac{x^2}{2}\right)$

$x(x^2+1) = x^3 + x$ ✓

80. $\boxed{\int_{\pi/3}^{x} \sec t \tan t \, dt} = \sec t \Big|_{\pi/3}^{x}$

$= \sec x - \sec \frac{\pi}{3}$ (by 1st FTC)

$= \boxed{\sec x - 2}$

$\int_{\pi/3}^{x} \sec t \tan t \, dt = \sec x - 2$

verify using 2nd FTC.

$\frac{d}{dx} \int_{\pi/3}^{x} \sec t \tan t \, dt = \frac{d}{dx}[\sec x - 2]$

$\sec x \tan x = \sec x \tan x$ ✓

86. $F(x) = \int_{0}^{x} \sec^3 t \, dt$

$F'(x) = \boxed{\sec^3 x}$

What about $F'(x)$ when $F(x) = \int_a^{g(x)} f(t)\,dt$?

Let $g(x) = u$

$$F'(x) = \frac{dF}{dx} = \frac{dF}{du} \cdot \frac{du}{dx} = \frac{d}{du}\left[F\right] \cdot \frac{du}{dx}$$

$$= \frac{d}{du}\left[\int_a^u f(t)\,dt\right] \cdot \frac{du}{dx} = f(u) \cdot \frac{du}{dx}$$

(i.e. we have chain rule)

HW

4.4: 45-51 odd

75-91 odd