4.4

47. $f(x) = 2\sec^2 x \quad , \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$

$\int_{-\pi/4}^{\pi/4} 2\sec^2 x\,dx = 2\sec^2 c\cdot\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right)$

$2\tan x \Big|_{-\pi/4}^{\pi/4} = \pi\sec^2 c$

$2 - (-2) = \pi\sec^2 c$

$\frac{4}{\pi} = \sec^2 c$

$\pm\frac{2}{\sqrt{\pi}} = \sec c$

$\pm\frac{\sqrt{\pi}}{2} = \cos c$

$c = \cos^{-1}\left(\pm\frac{\sqrt{\pi}}{2}\right)$

The Second Fundamental Theorem of Calculus:

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

What about $F'(x)$ when $F(x) = \int_a^{g(x)} f(t)\,dt$?

Let $g(x) = u$

$F'(x) = \frac{dF}{dx} = \frac{dF}{du}\cdot\frac{du}{dx} = \frac{d}{du}[F]\cdot\frac{du}{dx}$

$= \frac{d}{du}\left[\int_a^u f(t)\,dt\right]\cdot\frac{du}{dx} = f(u)\cdot\frac{du}{dx}$

(i.e. we have chain rule)

.

$$\boxed{\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)}$$

90. $F(x) = \int_2^{x^2} \frac{1}{t^3}\,dt = \int_2^{u} \frac{1}{t^3}\,dt$

Let $u = x^2$

$\frac{du}{dx} = 2x$

$$F'(x) = \frac{dF}{dx} = \frac{dF}{du}\cdot\frac{du}{dx} = \frac{d}{du}\left[F(x)\right]\cdot\frac{du}{dx}$$

$$= \frac{d}{du}\left[\int_2^u \frac{1}{t^3}\,dt\right]\cdot\frac{du}{dx}$$

$$= \frac{1}{u^3}\cdot\frac{du}{dx} = \frac{1}{(x^2)^3}\cdot 2x = \frac{2x}{x^6} = \boxed{\frac{2}{x^5}}$$

92. $F(x) = \int_0^{x^2} \sin \theta^2 \, d\theta$

$$F'(x) = \sin (x^2)^2 \cdot (x^2)'$$

$$= \boxed{2x \sin x^4}$$

88. $F(x) = \int_{-x}^{x} t^3 dt = \int_{-x}^{a} t^3 dt + \int_a^x t^3 dt$

$$= -\int_a^{-x} t^3 dt + \int_a^x t^3 dt$$

$$F'(x) = -\left[(-x)^3 \cdot (-1)\right] + x^3$$

$$= \boxed{0}$$

4.5 Integration by Substitution

12. $\int x^2(x^3+5)^4\,dx = \int (x^3+5)^4 \cdot x^2\,dx$

Let $u = x^3+5$

$\frac{du}{3} = \frac{\cancel{3}x^2\,dx}{\cancel{3}}$

$= \int u^4\left(\frac{1}{3}du\right)$

$= \frac{1}{15}u^5 + C$

$= \boxed{\frac{(x^3+5)^5}{15} + C}$

22. $\int \frac{x^2}{(16-x^3)^2}\,dx = \int \frac{-du}{3u^2}$

$u = 16-x^3$

$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$= \int -\frac{1}{3}u^{-2}\,du$

$= \frac{1}{3}u^{-1} + C$

$= \boxed{\frac{1}{3(16-x^3)} + C}$

4.5

50. $\int \sqrt{\tan x}\,\sec^2 x\,dx = \int u^{1/2}\,du$

$u = \tan x$

$du = \sec^2 x\,dx$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}(\tan x)^{3/2} + C}$

54. $\int \csc^2\left(\frac{x}{2}\right)dx = \int 2\csc^2 u\,du$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2du = dx$

$= -2\cot u + C$

$= \boxed{-2\cot\frac{x}{2} + C}$

52. $\int \frac{\sin x}{\cos^3 x}dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$\int -\frac{du}{u^3} = \int -u^{-3}du = \frac{1}{2}u^{-2} + C_1$

$= \frac{1}{2\cos^2 x} + C_1$

$\int \tan x \sec^2 x\,dx = \int u\,du = \frac{1}{2}u^2 + C_2$

$u = \tan x$

$du = \sec^2 x\,dx$

$= \frac{1}{2}\tan^2 x + C_2$

$\frac{\sin^2 x}{2\cos^2 x} = \frac{1 - \cos^2 x}{2\cos^2 x} = \frac{1}{2\cos^2 x} - \frac{1}{2} + C_2$

58. $\int x\sqrt{2x+1}\,dx = \int \frac{u-1}{2} \cdot u^{1/2} \cdot \frac{du}{2} =$

$u = 2x+1 \Rightarrow \frac{u-1}{2} = x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \int \left(\frac{1}{4}u^{3/2} - \frac{1}{4}u^{1/2}\right)du$

$= \frac{1}{10}u^{5/2} - \frac{1}{6}u^{3/2} + C$

$= \boxed{\frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + C}$

62. $\int \frac{2x+1}{\sqrt{x+4}}\,dx = \int \frac{2(u-4)+1}{u^{1/2}}\,du$

$u = x+4 \Rightarrow x = u-4$

$du = dx$

$= \int \left(\frac{2u^1}{u^{1/2}} - \frac{7}{u^{1/2}}\right)du$

$= \int \left(2u^{1/2} - 7u^{-1/2}\right)du$

$= \boxed{\frac{4}{3}(x+4)^{3/2} - 14(x+4)^{1/2} + C}$

Definite Integrals

66. $\int_{-2}^{4} x^2(x^3+8)^2\,dx = \int_{x=-2}^{4} \frac{u^2}{3}\,du$

$u = x^3 + 8$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$= \frac{1}{9}u^3 \Big|_{x=-2}^{4}$

$= \frac{x^3+8}{9}\Big|_{-2}^{4} = \frac{4^3+8}{9} - 0$

$= \boxed{8}$

HW

4.4: 45-51 odd

75-91 odd

4.5 # 7-33, 41-53,

57-63 odd