4.5

53. $\int \cot^2 x \, dx$

$= \int (\csc^2 x - 1) dx$

$= \boxed{-\cot x - x + C}$

$\frac{\sin^2 x + \cos^2 x = 1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$$\int \frac{du}{u} = \ln |u| + K$$

$$* \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + K$$

$$\int \tan u \, du = -\ln |\cos u| + K$$

$$\int \cot u \, du = \ln |\sin u| + K$$

$$\int \sec u \, du = \ln |\sec u + \tan u| + K$$

$$\int \csc u \, du = \ln |\csc u - \cot u| + K$$

$\int \tan x \, dx$

$= \int \frac{\sin x \, dx}{\cos x}$

let $u = \cos x$

$du = -\sin x \, dx$

$\int \frac{-du}{u} = -\ln|u| + C$

$= -\ln|\cos x| + C$

$$\left[\log_a [f(x)]\right]' = \frac{1}{\ln a \cdot f(x)} \cdot f'(x)$$

5.2

11. $\displaystyle\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx = \int \frac{1}{3}\frac{du}{u} = \frac{1}{3}\ln|u| + C$

$u = x^3+3x^2+9x$

$du = (3x^2+6x+9)\,dx$

$\frac{1}{3}du = (x^2+2x+3)\,dx$

$$= \boxed{\frac{1}{3}\ln|x^3+3x^2+9x| + C}$$

$\int u^{-1}du = \frac{u^0}{0}$ oops!

7. $\int \frac{x}{x^2+1}dx = \int \frac{1}{2}\frac{du}{u} = \boxed{\frac{1}{2}\ln|x^2+1| + C}$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

9. $\int \frac{x^2-4}{x}dx = \int \left(\frac{x^2}{x} - \frac{4}{x}\right)dx = \int \left(x - \frac{4}{x}\right)dx$

$= \boxed{\frac{x^2}{2} - 4\ln|x| + C}$

5.2

34. $\int \frac{\csc^2 t}{\cot t}dt = \boxed{-\ln|\cot t| + C}$

$u = \cot t$

$du = -\csc^2 t\,dt$

$\int -\frac{du}{u} = -\ln|u| + C$

64. $F(x) = \int_1^{x^2} \frac{1}{t}\, dt$

Find $F'(x)$.

$F(x) = \int_a^{g(x)} f(t)\, dt$

$F'(x) = f(g(x)) \cdot g'(x)$

$F'(x) = \frac{1}{x^2} \cdot (x^2)' = \frac{2x}{x^2} = \boxed{\frac{2}{x}}$

$F(x) = \ln|t| \Big|_1^{x^2} = \ln|x^2| - \ln|1| = \ln(x^2)$

$F'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$ ✓

5.4

$\int e^x dx = e^x + C$

$[e^x]' = e^x$

5.5

$[a^x]' = a^x \cdot \ln a$

$\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$

5.4

94. $\int \frac{e^{1/x^2}}{x^3}\,dx = \int e^{1/x^2} \cdot \frac{dx}{x^3} = \int -\frac{1}{2} e^u\,du$

$u = \frac{1}{x^2} = x^{-2}$

$du = -2x^{-3}\,dx$

$\frac{du}{-2} = \frac{\cancel{-2}dx}{x^3}$

$= -\frac{1}{2}e^u + C$

$= \boxed{-\frac{1}{2}e^{1/x^2} + C}$

104. $\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx$

$= \int \left(e^x + 2 + \frac{1}{e^x}\right) dx$

$= \int e^x\,dx + \int 2\,dx + \int \frac{dx}{e^x}$

$= e^x + 2x + \int \frac{dx}{e^x}$

$= e^x + 2x + \int e^{-x}\,dx$

$u = -x$

$du = -dx$

$-du = dx$

$\int -e^u\,du$

$= \boxed{e^x + 2x - e^{-x} + C}$

108. $\int \ln(e^{2x-1})\,dx$

$= \int (2x-1)\,dx$

$= \boxed{x^2 - x + C}$

5.5

68. $\int 2^{\sin x} \cos x\,dx = \int 2^u\,du$

$u = \sin x$

$du = \cos x\,dx$

$= \frac{1}{\ln 2} \cdot 2^u + C$

$= \boxed{\frac{2^{\sin x}}{\ln 2} + C}$

64. $\int_{-2}^{0} (3^3 - 5^2)\,dx$

$\quad\;\; 27 \quad 25$

$= \int_{-2}^{0} 2\,dx = 2x\Big|_{-2}^{0} = 2(0) - 2(-2)$

$= \boxed{4}$

5.4

102. $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\,dx$

$= \int \frac{2e^x - 2e^{-x}}{e^{2x} + 2e^x e^{-x} + e^{-2x}}\,dx$

$= \int \frac{2e^x - 2e^{-x}}{e^{2x} + e^{-2x} + 2}\,dx$

$u = e^{2x} + e^{-2x} + 2$

$du = (2e^{2x} - 2e^{-2x})\,dx$

5.4

102. $$\int \frac{2e^{x}-2e^{-x}}{(e^{x}+e^{-x})^{2}}\,dx = \int \frac{2\,du}{u^{2}}$$

$u = e^{x}+e^{-x}$

$du = (e^{x}-e^{-x})\,dx$

$2du = (2e^{x}-2e^{-x})\,dx$

$$= \int 2u^{-2}\,du$$

$$= -2u^{-1}+C$$

$$= \boxed{-\frac{2}{e^{x}+e^{-x}}+C}$$

<u>Homework #3</u>
4.4 #45-51 odd; 75-91 odd
4.5 #7-33 odd; 41-53 odd; 57-75 odd
5.2 #1-35 odd; 43-53 odd; 61, 63
5.4 #87-107 odd
5.5 #61-68 all