4.4

45. $f(x) = x - 2\sqrt{x}$, $[0, 2]$

$$\int_a^b f(x)\,dx = f(c) \cdot (b-a)$$

$$\frac{1}{2}x^2 - \frac{4}{3}x^{3/2}\Big|_0^2 = (c - 2\sqrt{c})(2)$$

$$x = 2x - 3\sqrt{x}$$

$$(3\sqrt{x})^2 = (\quad)^2$$

quadratic

4.5

57. $\int x\sqrt{x+2}\,dx = \int (u-2)\,u^{1/2}\,du$

Let $u = x+2$

$x = u - 2$

$du = dx$

$$= \int (u^{3/2} - 2u^{1/2})\,du$$

$$\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$$

$$\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

5.9 Inverse Trig Functions

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

2. $\displaystyle\int \frac{3dx}{\sqrt{1-4x^2}}$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$= \displaystyle\int \frac{3dx}{\sqrt{1^2-(2x)^2}}$ $u = 2x$

$\frac{3 \cdot du}{2} = \frac{\cancel{2}dx \cdot 3}{\cancel{2}}$

$= \displaystyle\int \frac{\frac{3}{2}du}{\sqrt{1^2-u^2}} = \frac{3}{2}\cdot \arcsin\frac{2x}{1} + C$

$= \boxed{\frac{3}{2}\arcsin 2x + C}$

8. $$\int_{\sqrt{3}}^{3} \frac{1}{9+x^2}\,dx$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$= \frac{1}{3}\arctan\frac{x}{3}\Big|_{\sqrt{3}}^{3}$$

$$= \frac{1}{3}\arctan(1) - \frac{1}{3}\arctan\left(\frac{\sqrt{3}}{3}\right)$$

$$= \frac{1}{3}\cdot\frac{\pi}{4} - \frac{1}{3}\cdot\frac{\pi}{6} = \boxed{\frac{\pi}{36}}$$

12. $$\int \frac{x^4-1}{x^2+1}\,dx$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$= \int \frac{(x^2-1)\cancel{(x^2+1)}}{\cancel{x^2+1}}\,dx = \int (x^2-1)\,dx = \boxed{\frac{1}{3}x^3 - x + C}$$

16. $$\int \frac{1}{x\sqrt{x^4-4}}\,dx$$

$u = x^2$

$$\frac{du}{2u} = \frac{2x\,dx}{2x^2}$$

$$\frac{du}{2u} = \frac{dx}{x}$$

$$= \frac{1}{2}\int \frac{du}{u\sqrt{u^2-2^2}}$$

$$= \boxed{\frac{1}{4}\operatorname{arcsec}\frac{x^2}{2} + C}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

30. $\displaystyle\int \frac{x-2}{(x+1)^2+4}\,dx$

Let $u = x+1$
$x = u-1$
$du = dx$

$$= \int \frac{u-1-2}{u^2+2^2}\,du = \int \frac{u-3}{u^2+2^2}\,du$$

$$= \int \frac{u\,du}{u^2+4} - 3\int \frac{du}{u^2+4} =$$

$v = u^2+4$
$dv = 2u\,du$
$\frac{dv}{2} = u\,du$

$$= \frac{1}{2}\int \frac{dv}{v} - \frac{3}{2}\arctan\frac{x+1}{2} + C$$

$$= \frac{1}{2}\ln|v| - \sim\sim\sim$$

$$= \frac{1}{2}\ln|u^2+4| - \sim\sim$$

$$= \boxed{\frac{1}{2}\ln|(x+1)^2+4| - \frac{3}{2}\arctan\frac{x+1}{2} + C}$$

32. $\displaystyle\int_{-2}^{2} \frac{dx}{x^2+4x+13}$ $\qquad\displaystyle\int \frac{du}{a^2+u^2}$

completing the square:

$$ax^2+bx+c \sim A(x-h)^2+K$$

$$a\left(x^2+\frac{b}{a}x\right) + c$$

$$a\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2\right) + c - \frac{b^2}{4a}$$

$$a\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$$

$$(a+b)^2 = a^2+2ab+b^2$$
$$(a-b)^2 = a^2-2ab+b^2$$

32. $\int_{-2}^{2} \frac{dx}{x^2+4x+13} = \int_{-2}^{2} \frac{dx}{(x+2)^2+9}$

$$\boxed{\begin{array}{l} x^2+4x+4+13-4 \\ (x+2)^2+9 \end{array}}$$

$= \frac{1}{3}\arctan \frac{x+2}{3}\Big|_{-2}^{2} =$

$= \frac{1}{3}\arctan\frac{4}{3} - \cancel{\frac{1}{3}\arctan 0}$ $=0$

$= \boxed{\frac{1}{3}\arctan\frac{4}{3}}$

42. $\int \frac{x}{\sqrt{9+8x^2-x^4}}\,dx$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+c$$

$9+8x^2-x^4 = \qquad -(x \quad)^2$

$9-(x^4-8x^2+16)+16$

$\int \frac{x\,dx}{\sqrt{25-(x^2-4)^2}} =$ $u=x^2-4$

$a=5$ . . .

Homework #3
4.4 #45-51 odd; 75-91 odd
4.5 #7-33 odd; 41-53 odd; 57-75 odd
5.2 #1-35 odd; 43-53 odd; 61, 63
5.4 #87-107 odd
5.5 #61-68 all
5.9 #1-41 odd
Take-home quiz
Ch 5 Review pp.405-407 #17-24, 49-56,71-72, 99-106