5.9 Inverse Trig Functions

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

5.2

27. $\displaystyle\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx = \int \frac{u+3}{u}\cdot 2(u+3)\,du$

$u = \sqrt{x} - 3 \Rightarrow u+3 = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2du = \frac{dx}{\sqrt{x}}$

$2\sqrt{x}\,du = dx$

$2(u+3)du = dx$

$$= \int \frac{2u^2+12u+18}{u}\,du$$

$$= \int 2u\,du + \int 12\,du + \int \frac{18}{u}\,du$$

$$= u^2 + 12u + 18\ln|u| + c$$

$$= (\sqrt{x}-3)^2 + 12(\sqrt{x}-3) + 18\ln\left|\sqrt{x}-3\right| + C$$

5.2

51. $\int \tan x = -\ln|\cos x| + c$

$\int \tan x = \ln|\sec x| + c$

$\log a^p = p \log a$

$\ln|\sec x| = \ln|(\cos x)^{-1}| = -\ln|\cos x|$

5.2

27. $\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx = \int \frac{(\sqrt{x}-3)+3}{\sqrt{x}-3}\,dx$

$u+3=\sqrt{x}$

$u=\sqrt{x}-3$

$du=\frac{1}{2\sqrt{x}}\,dx$

$2(u+3)\,du = dx$

$= \int \left(1 + \frac{3}{\sqrt{x}-3}\right) dx$

$= x + \int \frac{6u+12}{u}\,du$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

40. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$

$x^2-2x+1-1$

$$= \int \frac{dx}{(x-1)\sqrt{(x-1)^2-1}} = \boxed{\operatorname{arcsec}|x-1| + C}$$

$u = x-1$

$du = dx$

$a = 1$

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$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

100. $\int \frac{1}{3+25x^2}\,dx$

$$= \int \frac{1}{(\sqrt{3})^2+(5x)^2}\,dx$$

$$= \boxed{\frac{1}{\sqrt{3}}\arctan\frac{5x}{\sqrt{3}} + C}$$

104. $\displaystyle\int \frac{4-x\,dx}{\sqrt{4-x^2}}$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$= \underbrace{\int \frac{4\,dx}{\sqrt{4-x^2}}}_{} + \int \frac{-x\,dx}{\sqrt{4-x^2}}$$

$u = x \quad a = 2$

$du = dx$

$u = 4 - x^2$

$du = \cancel{2}x\,dx$

$$4\arcsin\frac{x}{2} + \int \frac{du}{2\sqrt{u}}$$

$$= 4\arcsin\frac{x}{2} + \int \frac{1}{2}u^{-1/2}\,du$$

$$= 4\arcsin\frac{x}{2} + u^{1/2} + C$$

$$= \boxed{4\arcsin\frac{x}{2} + \sqrt{4-x^2} + C}$$

105. $\displaystyle\int \frac{\arctan(x/2)}{4+x^2}\,dx$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

$u = \arctan\frac{x}{2}$

$$du = \frac{1}{1+\left(\frac{x}{2}\right)^2}\cdot\frac{1}{2}\,dx$$

$$\frac{du}{2} = \frac{\cancel{2}\,dx}{4+x^2}$$

$$\int \frac{1}{2}u\,du$$

$$= \frac{1}{4}u^2 + C$$

$$= \boxed{\frac{1}{4}\left(\arctan\frac{x}{2}\right)^2 + C}$$

Homework #3
4.4 #45-51 odd; 75-91 odd
4.5 #7-33 odd; 41-53 odd; 57-75 odd
5.2 #1-35 odd; 43-53 odd; 61, 63
5.4 #87-107 odd
5.5 #61-68 all
5.9 #1-41 odd
Ch 5 Review pp.405-407 #17-24, 49-56,71-72, 99-106

due Tues.

Take-home quiz due Wednesday

Test Friday? 12/13